Influence of Geometric Imperfections and In-Plane Constraints on Nonlinear Vibrations of Simply Supported Cylindrical Panels

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This papers deals with the effects of initial geometric imperfections on large-amplitude vibrations of cylindrical panels simply supported along all four edges. In-plane movable and in-plane immovable boundary conditions are considered for each pair of parallel edges. Depending on whether the number of axial and circumferential half waves are odd or even, the presence of geometric imperfections (taken to be of the same shape as the vibration mode) of the order of the shell thickness may significantly raise or lower the linear vibration frequencies. In general, an increase (decrease) in the linear vibration frequency corresponds to a more pronounced soft-spring (hard-spring) behavior in nonlinear vibration.

1 Introduction

The first investigation of the large-amplitude vibration of simply supported circular cylindrical shells was performed by Reissner [1]. Subsequent results by Chu [2] and Nowinski [3] show that the nonlinearity of this vibration problem was always of the hardening type. Evensen and Fulton [4] and Evensen [5] demonstrated that the nonlinearity may be either of the hardening or softening type depending on Evensen's aspect ratio and Evensen's nonlinearity parameter. They based their argument on the fact that the earlier results [1-3] failed to satisfy the singlevaluedness requirement of the circumferential displacement and they demonstrated the need to include the driven mode as well as the companion mode [4-6]. Their findings were qualitatively supported by experiments [7, 8] and an excellent summary of the historical developments (which includes the works of M. D. Olson, E. H. Dowell and C. S. Ventres, S. Atluri and J. H. Ginsberg, etc.) can be found in a review paper by Evensen [9].

The large-amplitude vibrations of open cylindrical panels were examined by Reissner [1] and followed by Cummings [10, 11]. It was found that the nonlinearity of this vibration problem can be classified as soft-spring or hard-spring type. Extension of the work to include the effects of nonlinear elastic foundations were examined [12, 13].

The effects of geometric imperfections on the large-amplitude vibrations of rectangular plates, circular plates, and spherical shells have been examined in the author's earlier papers [14-16]. The effects of imperfections on nonlinear vibrations of closed cylindrical shells have also been investigated by Watawala and Nash [17]. However, the effects of geometric imperfections on large-amplitude vibrations of open cylindrical shells simply supported at all four edges have not been examined. Further, the influence of various types of in-plane boundary conditions (which have been demonstrated to be of major concern in linear-free vibrations of cylindrical shells [18]) have also not been studied. Since the cylindrical panels are open, the singlevaluedness requriements of the displacements need not and in fact, should not be enforced.

The present analysis is based on a solution of the nonlinear Donnell-type dynamic equilibrium and compatibility differential equations for a cylindrical panel written in terms of a stress function and an out-of-plane displacement. The geometric imperfections are taken to be of the same spatial shape as the vibration mode. Based on the assumed sinusoidal vibration mode shape, the stress function that satisfies the nonlinear compatibility equation exactly is sought. The nonlinear dynamic equilibrium equation is then satisfied approximately using the Galerkin procedure.

Depending on whether the number of axial and circumferential half-waves are odd or even, the presence of geometric imperfection may significantly raise or lower the

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linear vibration frequencies. Furthermore, the increase (decrease) of the linear vibration frequency generally corresponds to a more pronounced soft-spring (hard-spring) nonlinear vibration behavior. In the special case of perfect cylindrical panels (zero imperfection amplitude) it is found that the in-plane boundary conditions do not affect the linear frequencies, even though they may have a significant influence on the nonlinear hard-spring or soft-spring behaviors. The parameter variations involving the flatness parameter (Koiter [19] and Hui et al. [20]), the length to panel circumferential width ratio, and the number of axial and circumferential half-waves are examined.

2 Analysis

The dynamic analogue of the nonlinear Donnell-type differential equations for cylindrical shells written in terms of the out-of-plane displacement (positive outward) W and the stress function F, incorporating the possibility of the presence of geometric imperfections W_0 are (see [21] among others)

$$[(1+i\eta)Eh^3/(4c^2)](W,_{XXXX}+W,_{YYYY}+2W,_{XXYY})$$
$$+(1/R)(F,_{XX})=Q(X,Y,\bar{t})-\rho W,_{\bar{t}\bar{t}}+F,_{YY}(W+W_0),_{XX}$$
$$+F,_{XX}(W+W_0),_{YY}-2F,_{XY}(W+W_0),_{XY} \quad (1)$$

$$\{1/[Eh(1+i\eta)]\}(F,_{XXXX}+F,_{YYYY}+2F,_{XXYY})$$
$$=(1/R)(W,_{XX})+[(W,_{XY})^2+2W_{0,XY}W,_{XY}$$
$$-(W+W_0),_{XX}W,_{YY}-W_{0,YY}W,_{XX}] \quad (2)$$

where $c=[3(1-\nu^2)]^{1/2}$, ν is Poisson's ratio, X and Y are the axial and circumferential coordinates, R is the shell radius, h is the thickness, E is Young's modulus, ρ is the shell mass per unit area, $Q(X,Y,\bar{t})$ is the forcing function, $\bar{t}$ is the time, $i=(-1)^{1/2}$, and η is the loss factor associated with the complex modulus model for structural damping.

Introducing the nondimensional quantities ($q_0 = (2cR/h)^{1/2}$),

$$(w,w_0)=(W,W_0)/h, \quad f=2cF/(Eh)^3),$$
$$(x,y)=(q_0/R)(X,Y)$$
$$q(x,y,t)=[2R^2/(Eh^2)]Q(X,Y,\bar{t}), \quad t=\omega_r\bar{t},$$
$$(\omega_r)^2=Eh/(2\rho R^2) \quad (3)$$

where ω_r is the reference frequency, the nondimensional nonlinear dynamic equilibrium and compatibility equations for cylindrical panels become

$$(1+i\eta)(w,_{xxxx}+w,_{yyyy}+2w,_{xxyy})+f,_{xx}$$
$$=(1/2)q(x,y,t)-(1/2)w,_{tt} \quad (5)$$
$$+(2c)[f,_{xx}(w+w_0),_{yy}+f,_{yy}(w+w_0),_{xx}-2f,_{xy}(w+w_0),_{xy}]$$

$$[1/(1+i\eta)](f,_{xxxx}+f,_{yyyy}+2f,_{xxyy})$$
$$=w,_{xx}+(2c)[(w+2w_0),_{xy}w,_{xy}$$
$$-(w+w_0),_{xx}w,_{yy}-w_{0,yy}w,_{xx}] \quad (6)$$

The vibration mode, the initial goemetric imperfections, and the forcing function are assumed to have the same spatial distribution. The cylindrical panels are simply supported at all four edges so that [14],

$$[w(x,y,t),\ w_0(x,y),\ q(x,y,t)]$$
$$=[w(t),\ \mu,\ q(t)]\sin(Mx)\sin(Ny) \quad (7)$$

where $w(t)$ and μ are the vibration and imperfection amplitudes normalized with respect to the shell thickness. Furthermore, the axial and circumferential nondimensional wave numbers M and N are defined to be (the symbol n is never used in this paper in order to avoid confusion with the number of circumferential full-waves in a closed circular cylindrical shell),

$$M=m\pi R/(Lq_0)=md/(2\theta L), \quad N=\bar{n}/(2\theta), \quad \theta=q_0 d/(2\pi R) \quad (8)$$

In the foregoing, θ is the flatness parameter, d is the width of the cylindrical panel defined by the circumferential curved distance between the two longitudinal edges, L is the shell length, and the integers m and $\bar{n}$ are the number of half-waves in the longitudinal ($X=0$ to L) and circumferential ($Y=0$ to d) directions, respectively. The flatness parameter is introduced in order to lump the effects of the radius to thickness ratio and the radius to panel width ratio into a single parameter.

It will be shown from the computation of the definite integrals that is is necessary to specify the integers m and $\bar{n}$ in addition to the specification of the wave numbers M and N. Alternatively, one may specify m, $\bar{n}$, θ, and L/d. To compare the results for cylindrical panels with that of a complete circular cylindrical shell, it is convenient to define Evensen's nonlinearity parameter ϵ and Evensen's aspect ratio ξ in terms of the present notation.

$$\delta(\text{Evensen})=4c^2N^4=c\bar{n}^2/\theta^2$$
$$\xi(\text{Evensen})=M/N=md/(L\bar{n}) \quad (9)$$

The in-plane boundary conditions at the two curved edges ($x=0$ and $x=q_0L/R$) are

$$f,_{yy}=0 \quad \text{or} \quad u=0$$
$$f,_{xy}=0 \quad \text{or} \quad v=0 \quad (10)$$

while at the two longitudinal edges $y=0$ and $y=2\pi\theta$, they are,

$$f,_{xy}=0 \quad \text{or} \quad u=0$$
$$f,_{xx}=0 \quad \text{or} \quad v=0 \quad (11)$$

Since the mixed formulation is employed, the displacement boundary conditions can only be satisfied in the average (see for example [7, 22]) among others. In-plane boundary conditions type 1, 2, 3, and 4 are considered in this paper (Appendix A).

Substituting $w_0(x.y)$ and $w(x,y,t)$ into equation (6), the stress function that satisfies the nonlinear compatibility equation exactly is,

$$f(x,y,t)=(1+i\eta)\{c_0w(t)\sin(Mx)\sin(Ny)$$
$$+[w(t)^2+2\mu w(t)][c_1\cos(2Mx)+c_2\cos(2Ny)]$$
$$+E_1(t)(x^2/2)+E_2(t)(y^2/2)\} \quad (12)$$

where

$$c_0=\frac{-M^2}{(M^2+N^2)^2}, \quad c_1=(c/16)(N/M)^2, \quad c_2=(c/16)(M/N)^2$$
$$[E_1(t), E_2(t)]=(e_1e_2)[w(t)^2+2\mu u)(t)] \quad (13)$$

Finally, substituting $w_0(x,y)$, $w(x,y,t)$, and $f(x,y,t)$ into the nonlinear equilibrium equation and applying the Galerkin procedure (multiplying both sides by $\sin(Mx)\sin(Ny)$ and then integrating over the shell area), one obtains,

$$(1+i\eta)\{[(M^2+N^2)^2-c_0M^2]w(t)$$
$$+[w(t)^2+2\mu w(t)](4I_1H_1e_1)\}=(1/2)q(t)-(1/2)w(t),_{tt}$$
$$+(1+i\eta)[w(t)+\mu]\{(-d_1)[w(t)^2+2\mu w(t)]+d_2w(t)\} \quad (14)$$

where (d_1 and d_2 are independent of the geometric imperfection amplitude),

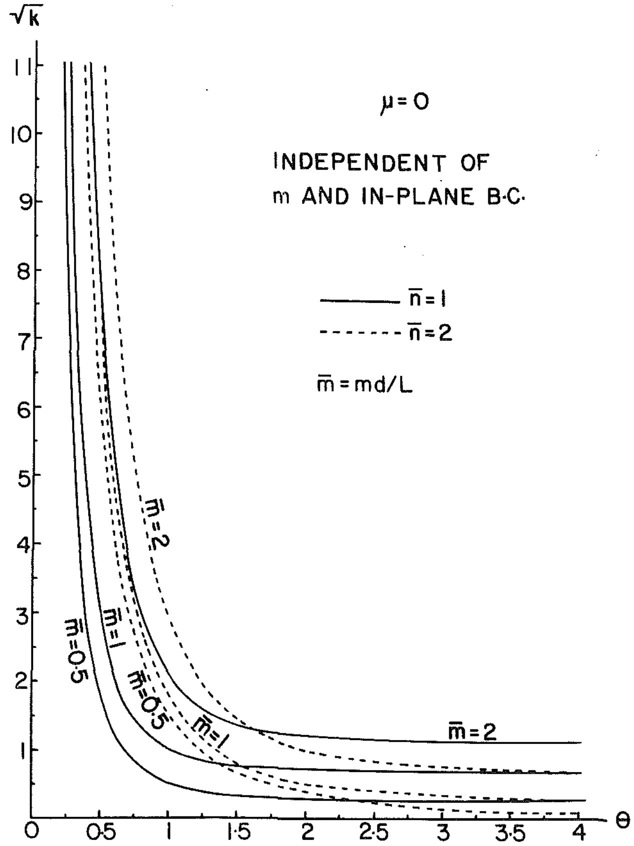

Fig. 1(*a*) Linear frequency ($k^{1/2} = \omega/\omega_r$) versus flatness parameter for perfect cylindrical panels

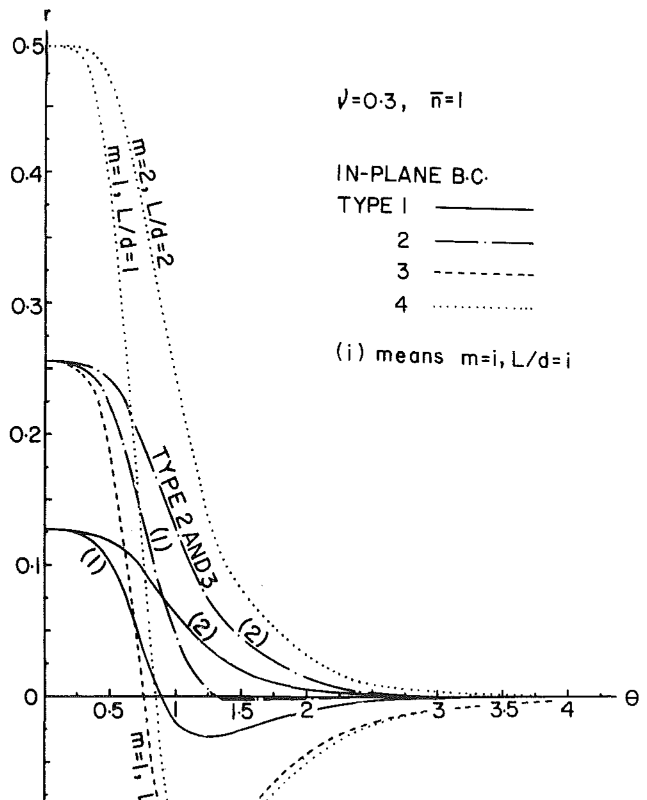

Fig. 1(*b*) Nonlinearity parameter versus flatness parameter for perfect cylindrical panels ($\bar{n} = 1$)

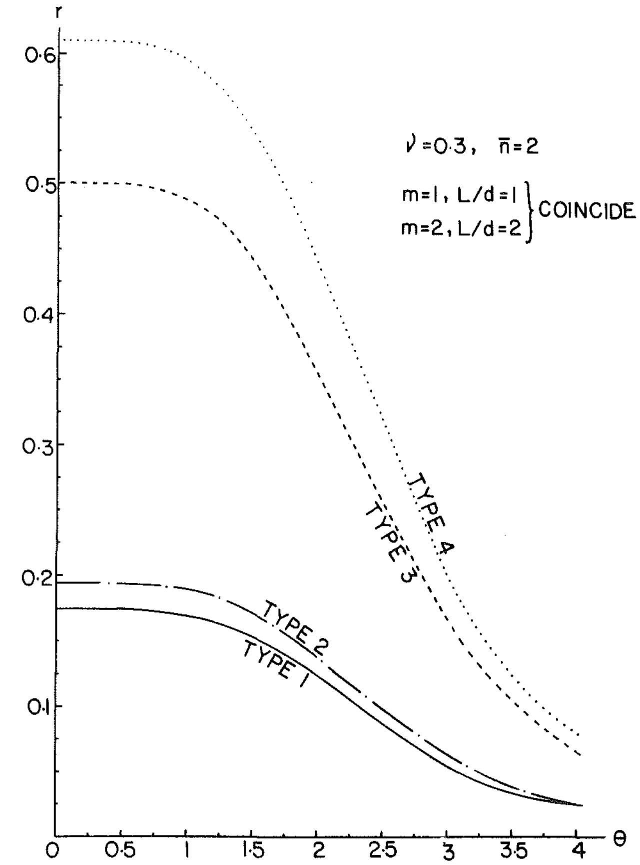

Fig. 1(*c*) Nonlinearity parameter versus flatness parameter for perfect cylindrical panels ($\bar{n} = 2$)

Fig. 1

$$d_1 = (2c)\,[N^2 e_1 + M^2 e_2 + 2M^2 N^2 (c_1 + c_2)]$$

$$d_2 = (2c)\,(8M^2 N^2 c_0)(I_2 H_2 - I_3 H_3) \tag{15}$$

In the foregoing, the integrals I_1, I_2, I_3, H_1, H_2, and H_3 are defined to be ($z = q_0 L/R$),

$$(I_1, I_2, I_3) = (1/z)\int_0^z [\sin(Mx), \sin^3(Mx), \sin(Mx)\cos^2(Mx)]\,dx$$

$$(H_1, H_2, H_3) = [1/(2\pi\theta)]\int_0^{2\pi\theta} [\sin(Ny), \sin^3(Ny), \sin(Ny)\cos^2(Ny)]\,dy \tag{16}$$

Thus, if m is odd, $I_1 = 2/(m\pi)$, $I_2 = 4/(3m\pi)$, and $I_3 = 2/(3m\pi)$ while if m is even, $I_1 = I_2 = I_3 = 0$. Further if $\bar{n}$ is odd, $H_1 = 2/(\bar{n}\pi)$, $H_2 = 4/(3\bar{n}\pi)$, and $H_3 = 2/(3\bar{n}\pi)$ while if $\bar{n}$ is even, $H_1 = H_2 = H_3 = 0$. Finally, if both m and $\bar{n}$ are odd,

$$I_1 H_1 = 4/(m\bar{n}\pi^2), \quad I_2 H_2 - I_3 H_3 = 4/(3m\bar{n}\pi^2),$$

$$d_2 = 64cM^2 N^2 c_0/(3m\bar{n}\pi^2) \tag{17}$$

and otherwise, $I_1 H_1 = 0$, $I_2 H_2 - I_3 H_3 = 0$, and $d_2 = 0$. The quantities e_1 and e_2 are constants that depend on the in-plane boundary conditions and they are defined in Appendix A.

The preceding nonlinear ordinary differential equation in time can be written in terms of the well-known Duffing's equation with an additional quadratic term in the standard form,

$$w(t)_{,tt} + [kw(t) + (\epsilon k a_2)\,w(t)^2 + (\epsilon k)\,w(t)^3](1 + i\eta) = q(t) \tag{18}$$

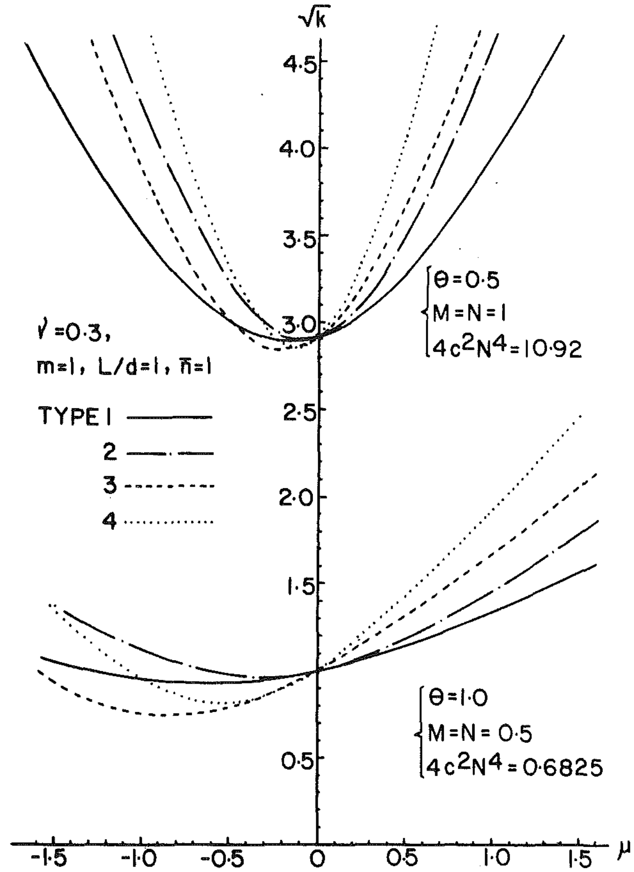

Fig. 2(*a*) Linear frequency ($k^{1/2} = \omega/\omega_r$) versus imperfection amplitude for θ = 0.5 and 1 (m = 1, L/d = 1)

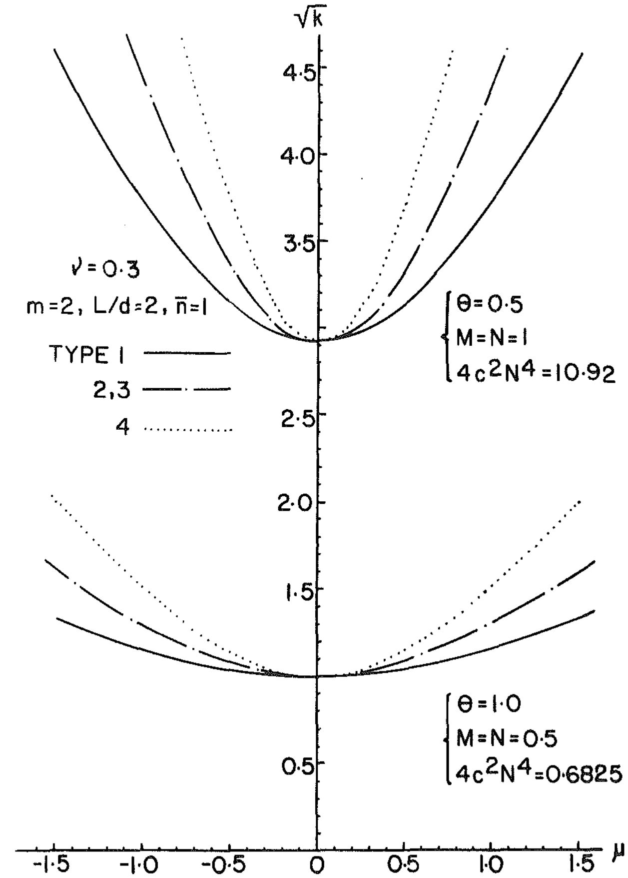

Fig. 2(*b*) Linear frequency ($k^{1/2} = \omega/\omega_r$) versus imperfection amplitude for θ = 0.5 and 1 (m = 2, L/d = 2)

Fig. 2

where

$$k = (2)\{(M^2+N^2)^2 + [M^4/(M^2+N^2)^2] + (2\mu^2 d_1 - \mu d_2) + 8\mu I_1 H_1 e_1\}$$

$$\epsilon = 2d_1/k, \quad a_2 = (3\mu d_1 - d_2 + 4I_1 H_1 e_1)/d_1 \tag{19}$$

In the special case of an infinitely long perfect cylindrical panel ($\mu = 0$, L/d = infinite, and $M = 0$), one obtains,

$$\epsilon k = 2N^4, \quad k = 4cN^2 e_1 \quad \text{and} \quad a_2 = 2I_1 H_1/(cN^2) \tag{20}$$

It should be noted that for a perfect cylindrical panel ($\mu = 0$), the linear vibration frequency defined by $k^{1/2}$ is independent of e_1 and e_2. That is, the linear vibration frequency is independent of the in-plane boundary conditions. Finally, since k in equations (18) and (19) are independent of the definite integrals I_1, I_2, I_3, H_1, H_2, and H_3, the linear vibration frequency is independent on whether the number of axial and circumferential half-waves are odd or even.

Assuming that the forcing function is periodic such that ($\Omega = \omega/\omega_r$)

$$q(t) = q_1 \cos(\omega\bar{t}) = q_1 \cos(\Omega t) \tag{21}$$

the solution of the linearized (neglecting the quadratic and cubic terms) differential equation is,

$$w(t) = A\cos(\Omega t) \tag{23}$$

where the absolute value of the complex quantity A is,

$$|A| = \frac{(q_1/k)}{\{[1-(\Omega^2/k)]^2 + \eta^2\}^{1/2}} \tag{23}$$

The backbone curves for large-amplitude free vibrations of simply supported cylindrical panels with no damping can be computed by solving the Duffing-type equation using Linstedt's perturbation method. It follows that the ratio of the nonlinear to the linear vibration frequency is related to the vibration amplitude A by (see the Appendix of [15] and note that A is measured from the deformed, static state),

$$\Omega/\Omega_0 = 1 + rA^2 - (15\,\epsilon^2 A^4/256) \tag{24}$$

where

$$r = (3\epsilon/8) - (5a_2{}^2\epsilon^2/12) = (3\epsilon/8)[1 - (10a_2{}^2\epsilon/9)] \tag{25}$$

Thus, at least for small values of the vibration amplitude A, the sign of the nonlinearity parameter r (not to be confused with Evensen's nonlinearity parameter defined in [4, 5]) determines whether the large-amplitude vibration of cylindrical panels is of the hardening or softening type. A positive value of r indicates hard-spring behavior while a negative value of r indicates soft-spring behavior. A larger positive value of r indicates a more pronounced hard-spring behavior.

3 Results and Discussions

Figure 1(*a*) shows a plot of the linear vibration frequency of perfect cylindrical panels (defined to be $k^{1/2} = \omega/\omega_r$) versus the flatness parameter (defined in equation (8)) for various values of md/L and $\bar{n}$. Note that the results for the perfect panels are independent of the in-plane boundary conditions as well as independent of whether the number of axial and circumferential half-waves are odd or even. The expression for k in equation (18) can be rewritten in the form ($\bar{m} = md/L$).

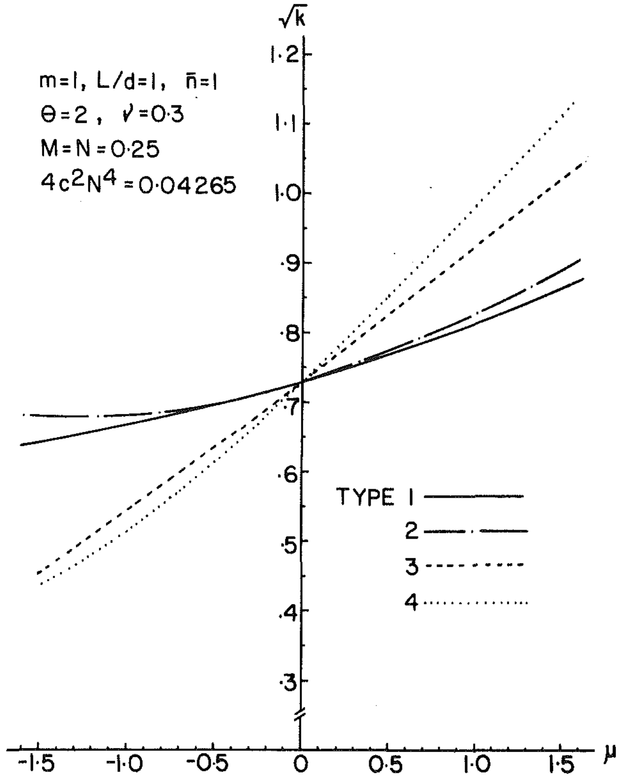

Fig. 3 Linear frequency ($k^{1/2} = \omega/\omega_r$) versus imperfection amplitude for $\theta = 2$ ($m = 1$, $L/d = 1$)

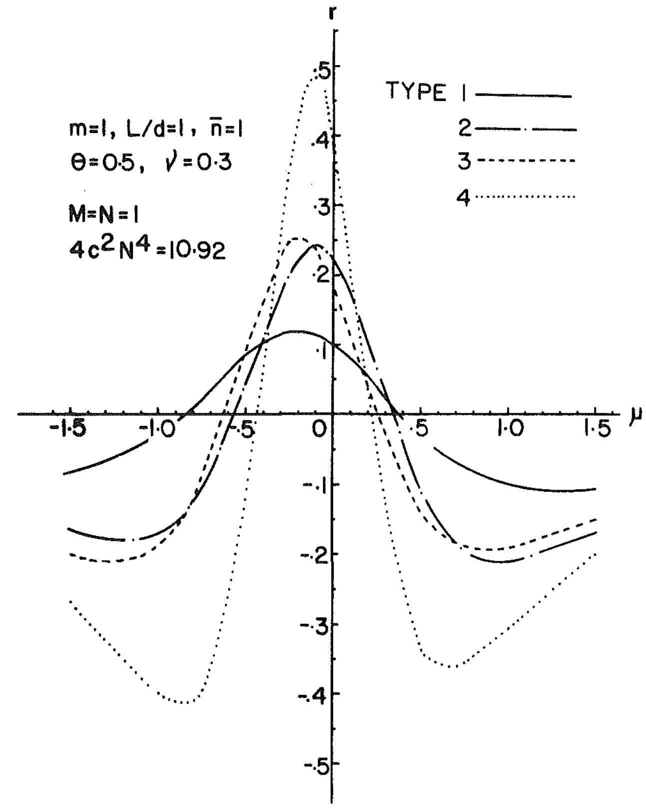

Fig. 4 Nonlinearity parameter versus imperfection amplitude for $\theta = 0.5$ and $\bar{n} = 1$ ($m = 1$, $L/d = 1$)

$$k=(2)\{[(\bar{m}^2+\bar{n}^2)^2/(16\theta^4)]+[\bar{m}^4/(\bar{m}^2+\bar{n}^2)^2]\} \tag{26}$$

where it may be observed that the term involving θ^4 will rapidly become negligible for $\theta \geq 4$.

The corresponding nonlinearity parameter r is shown in Fig. 1(b) for each of the four types of in-plane boundary conditions, keeping $\bar{n}=1$. For a fixed value of the flatness parameter θ and for each type of in-plane boundary condition, it can be seen that the $m=2$, $L/d=2$ curves lie above the $m=1$, $L/d=1$ curves even though they both imply $md/L=1$ (that is, the same value M). For a large value of the flatness parameter θ, the nonlinearity parameter approaches zero so that the nonlinear frequency is more or less the same as the linear frequency (at least for small-vibration amplitude). On the other hand, for sufficiently small values of θ, the nonlinearity parameter approaches the flat plate limit ($\theta=0$) with zero slope. As noted earlier, a positive value of the nonlinearity parameter r denotes hard-spring behavior while a negative value of r denotes soft-spring behavior.

Figure 1(c) shows a plot of the nonlinearity parameter r versus the flatness parameter θ for perfect cylindrical panels for four types of in-plane boundary conditions, keeping $\bar{n}=2$. Here, for each type of in-plane boundary condition, the curves for $m=1$, $L/d=1$ coincide with the curves for $m=2$, $L/d=2$ because the product of the integrals I_1H_1, I_2H_2, and I_3H_3 vanishes for $\bar{n}=2$ regardless of whether m is odd or even. All four curves converge to zero for sufficiently large values of the flatness parameter θ.

Figure 2(a) shows a graph of the linear vibration frequency ($k^{1/2}=\omega/\omega_r$) versus the amplitude of the initial geometric imperfection μ for $\theta=0.5$ and 1.0, keeping $m=1$, $L/d=1$, and $\bar{n}=1$. Since the out-of-plane displacement is defined to be positive outward, it is clear that a positive value of the imperfection amplitude μ for the $m=1$, $\bar{n}=1$ case will further increase the shell curvature. On the other hand, a small negative value of μ will actually reduce the shell curvature since the perfect cylindrical panel has an outward curvature while the geometric imperfection has an inward curvature (the curvatures tend to cancel out). A sufficiently large negative value of μ means that the shell actually has an increasingly predominant inward curvature. From this figure, it can be seen that an increase (decrease) of the shell curvature caused by the presence of the geometric imperfection will also increase (decrease) the linear vibration frequency.

It is interesting to observe from Fig. 2(b) that in the case $m=2$, $L/d=2$, $\bar{n}=1$, the linear vibration frequency is independent of the sign of the imperfection amplitude μ. This is because the geometric imperfection is taken to be of the same shape as the vibration mode so that for a panel with two axial half-waves, the shell actually increases its curvature for half of the cylindrical panel (say from $X=0$ to $X=L/2$) whereas its curvature decreases for the remaining half of the panel.

In the special case when either the number of axial half-waves m or the number of circumferential half-waves $\bar{n}$ (or both) is even, the products of the definite integrals I_1H_1, I_2H_2, and I_3H_3 vanish. Thus, equations (17) become ($d_2=0$).

$$k=(2)\{(M^2+N^2)^2+[M^4/(M^2+N^2)^2]+(4c\mu^2)[N^2e_1+M^2e_2+2M^2N^2(c_1+c_2)]\}$$

$$\epsilon=[N^2e_1+M^2e_2+2M^2N^2(c_1+c_2)](4c/k) \tag{27}$$

$$a_2=3\mu$$

In the further special case when $M=N$, it may be observed that the quantity $N^2e_1+M^2e_2$ remains invariant for the in-plane boundary conditions type two and type three. Consequently, the linear frequency curves (as well as the nonlinearity parameter curves) for the type two boundary condition coincide with that for the type three boundary condition.

For a larger value of the flatness parameter $\theta=2$, it is clear from Fig. 3 that for the important special case $m=1$, $\bar{n}=1$, a negative value of the imperfection amplitude will reduce the shell curvature and thus, the linear frequency decreases. On

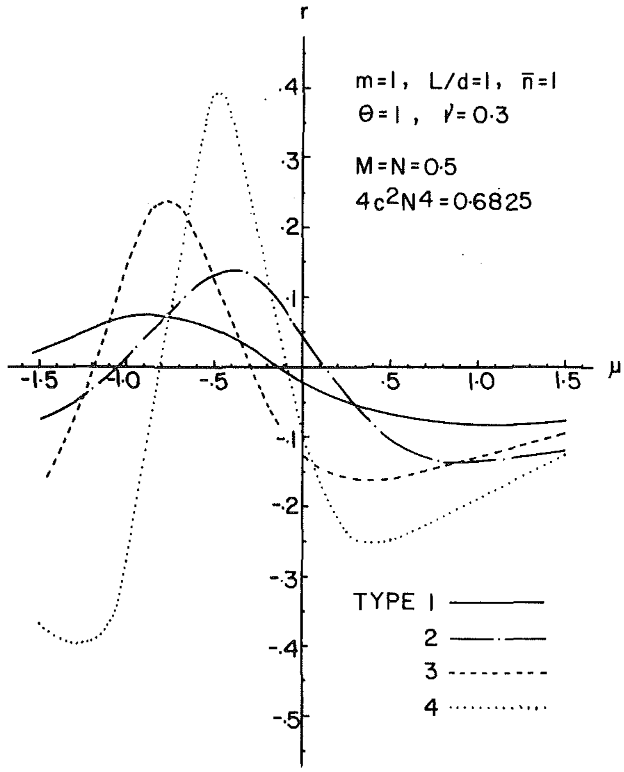

Fig. 5(a) Nonlinearity parameter versus imperfection amplitude for θ = 1 and $\bar{n}$ = 1 ($m=1, L/d=1$)

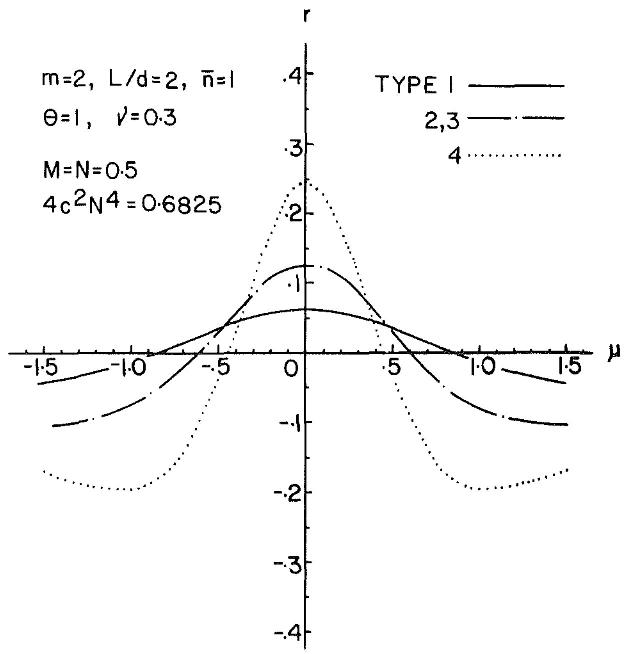

Fig. 5(b) Nonlinearity parameter versus imperfection amplitude for θ = 1 and $\bar{n}$ = 1 ($m=2, L/d=2$)

Fig. 5

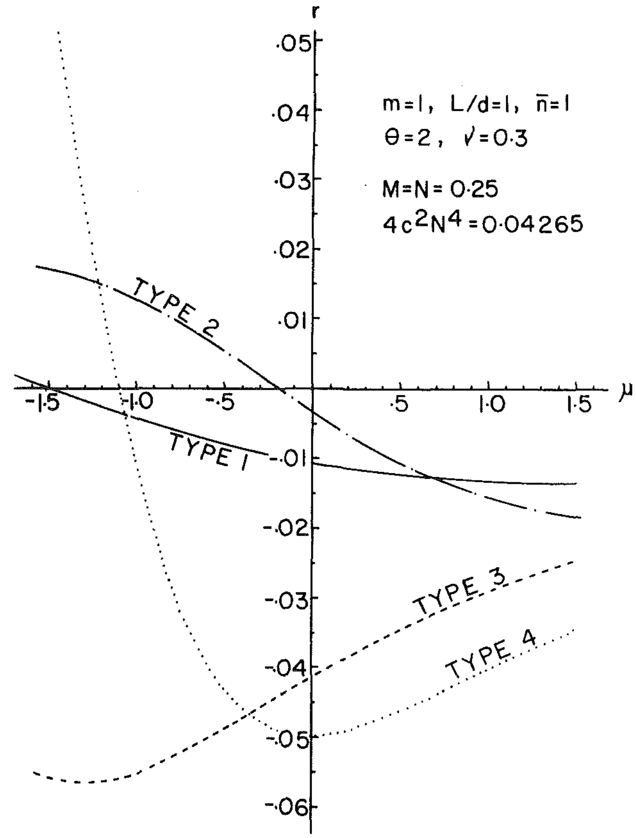

Fig. 6(a) Nonlinearity parameter versus imperfection amplitude for θ = 2 and $\bar{n}$ = 1 ($m=1, L/d=1$)

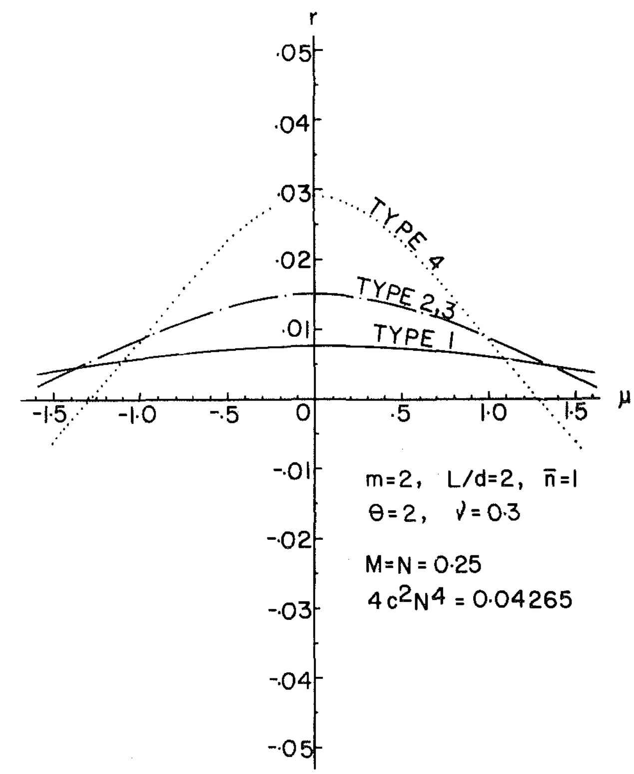

Fig. 6(b) Nonlinearity parameter versus imperfection amplitude for θ = 2 and $\bar{n}$ = 1 ($m=2, L/d=2$)

Fig. 6

the other hand, a positive value of μ will increase the curvature and thus, increase the linear frequency. For brevity, the corresponding curves for $m=2$, $L/d=2$, and $\bar{n}=1$ will not be shown. They are found to be independent of μ and they lie between $k^{1/2}=0.725$ and 0.85 for $-1.5 \leq \mu \leq 1.5$.

Figure 4 shows a graph of nonlinearity parameter r versus the imperfection amplitude μ for the flatness parameter $\theta=0.5$, $m=1$, $L/d=1$, and $\bar{n}=1$. Comparing the top sets of curves in Fig. 2(a) with Fig. 4, it can be seen that the minimum peak for the linear vibration frequency corresponds to the maximum peak for the nonlinearity parameter. In general, an increase in the linear vibration frequency (due to the presence of the geometric imperfection) is accompanied by a decrease in the value of the nonlinearity parameter r. The maximum peaks for the nonlinearity parameter versus flatness parameter curves corresponds to negative values of the imperfection amplitude and thus, the results depend on the sign of the imperfection amplitude. For brevity, the corresponding curves for $\theta=0.5$, $m=2$, $L/d=2$, and $\bar{n}=1$ will not be shown. Again, they are independent of the imperfection amplitude (symmetrical with respect to μ). The curves are shifted to the right with maximum peaks correspond to $\mu=0$.

Figure 5(a) shows a graph of the nonlinearity parameter versus the imperfection amplitude for $\theta=1$, $m=1$, $L/d=1$, and $\bar{n}=1$. Comparing Figs. 4(a) and 5, it can be seen that the maximum peaks are now shifted further to the left. This is because for large values of the flatness parameter, a larger negative value of the imperfection amplitude is needed to cause the shell to possess an inward curvature (the perfect cylindrical panel initially has an outward curvature). Again, comparing the lower sets of curves in Fig. 2(a) with Fig. 5(a) for $\theta=1$, an increase in the linear vibration frequency is associated with a decrease in the nonlinearity parameter. The curves for $\theta=1$, $m=2$, $L/d=2$, and $\bar{n}=1$ are shown in Fig. 5(b). These curves are symmetrical with respect to μ with maximum peak at $\mu=0$. Similar trends for $\theta=2$ are displaced in Figs. 6(a, b) and it can be seen that the $m=L/d=1$ and $m=L/d=2$ curves no longer resemble each other.

4 Concluding Remarks

The effects of initial geometric imperfections and four types of in-plane boundary conditions on the linear and nonlinear vibration behavior of cylindrical panels simply supported along all four edges have been examined. Keeping $\bar{n}=1$ and fixing the value of θ, it is found that for an imperfect cylindrical panel, the linear vibration frequency as well as the nonlinearity parameter for the two cases, ($m=1$, $L/d=1$) and ($m=2$, $L/d=2$) are different. In general, an increase (decrease) in the linear vibration frequency is accompanied by a decrease (increase) in the nonlinearity parameter.

Extension of the present work to large-amplitude vibrations of circular cylindrical shells, enforcing the exact simply supported and clamped boundary conditions as well as the exact singlevaluedness requirement of the circumferential displacement, is in progress.

Acknowledgment

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APPENDIX A

In-Plane Movable and In-Plane Immovable Boundary Conditions

To examine the in-plane boundary conditions in the mixed formulation, it is necessary to express the stress function F in terms of the axial, circumferential, and out-of-plane displacements U, V, and W in the form, (letting $\eta = 0$ and ϵ_x, ϵ_y, and ϵ_{xy} are the strains),

$$(1-\nu^2)(F,_{YY}) = (Eh)\,(\epsilon_x + \nu\epsilon_y)$$
$$(1-\nu^2)(F,_{XX}) = (Eh)\,(\nu\epsilon_x + \epsilon_y)$$
$$(1+\nu)\,(-F,_{XY}) = (Eh)\,(\epsilon_{xy}) \qquad (A1)$$

These relations imply,

$$F,_{YY} - \nu F,_{XX} = (\mathrm{E}h)\ \epsilon_x$$
$$F,_{XY} - \nu F,_{YY} = (\mathrm{E}h)\ \epsilon_y \qquad (A2)$$

Equations ($A1$) can be written in nondimensional form in terms of the displacements, ($U = hu/q_0$, $V = hv/q_0$),

$$(1-\nu^2)(f,_{yy}) = u,_x + (\nu)\,(v,_y + w) + (c)\,[(w+2w_0),_x w,_x + (\nu)\,(w+2w_0),_y w,_y]$$
$$(1-\nu^2)(f,_{xx}) = \nu u,_x + v,_y + w + (c)\,[\nu(w+2w_0),_x w,_x + (w+2w_0),_y w,_y] \qquad (A3)$$

Likewise equations ($A2$) can also be expressed in nondimensional form [20]

$$f,_{yy} - \nu f,_{xx} = u,_x + c(w+2w_0),_x w,_x$$
$$f,_{xx} - \nu f,_{yy} = v,_y + w + c(w+2w_0),_y w,_y \qquad (A4)$$

The four types of in-plane boundary conditions under consideration are,

Type 1 In-Plane Movable on all four edges.

$$e_1 = 0 \quad \text{and} \quad e_2 = 0 \tag{A5}$$

Type 2 Curved Edges Immovable and Longitudinal Edges Movable. Since the longitudinal edges are movable in the circumferential direction, it is clear that $f,_{xx} = 0$ at the two longitudinal edges. Furthermore,

$$f,_{yy} = (cM^2/4)\,[w(t)^2 + 2\mu w(t)] + \text{sinusoidal terms} \tag{A6}$$

so that

$$e_1 = 0, \quad e_2 = cM^2/4 \tag{A7}$$

Type 3 Curved Edges Movable and Longitudinal Edges Immovable. Since the two curved edges are movable, it may be concluded that $f,_{yy} = 0$ at these two curved edges. Furthermore,

$$f,_{xx} = (cN^2/4)\,[w(t)^2 + 2\mu w(t)] + \text{sinusoidal terms} \tag{A8}$$

so that

$$e_1 = cN^2/4, \quad e_2 = 0 \tag{A9}$$

Type 4 In-Plane Immovable on All Four Edges. Collecting only the constant terms in equation ($A2$) one obtains,

$$(1-\nu^2)f,_{yy} = (c/4)(M^2 + \nu N^2)\,[w(t)^2 + 2\mu w(t)] + \text{sinusoidal terms}$$

$$(1-\nu^2)f,_{xx} = (c/4)(\nu M^2 + N^2)\,[w(t)^2 + 2\mu w(t)] + \text{sinusoidal terms} \tag{A10}$$

so that,

$$e_1 = c(\nu M^2 + N^2)/[4(1-\nu^2)]$$

$$e_2 = c(M^2 + \nu N^2)/[4(1-\nu^2)] \tag{A11}$$