Fibre Science and Technology **21** (1984) 327–340

Shear Buckling of Anti-Symmetric Cross Ply Rectangular Plates

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SUMMARY

This paper deals with buckling of anti-symmetric cross ply, simply supported rectangular flat plates under shear loads. The effects of the Young's moduli ratios and the number of layers for these plates which exhibit bending–stretching coupling are examined. The linearized equilibrium and compatibility differential equations for buckling are solved, using the Galerkin procedure, by assuming a double sine series for the out-of-plane displacement. Both the symmetric and anti-symmetric buckling modes are considered. The asymmetric nature of the initial postbuckling problem is demonstrated by the existence of a non-zero cubic term of the potential energy within the context of Koiter's theory of elastic stability.

INTRODUCTION

Buckling of plates has been a subject which has attracted the attention of numerous researchers over the past several decades, since it is an important design consideration in many aerospace and mechanical engineering structures. In particular, shear buckling of rectangular isotropic homogeneous plates has been thoroughly analysed by Southwell and Skan,[1] Seydel,[2] Stein and Neff,[3] Batdorf and Stein[4] and Budiansky *et al.*,[5] among others. Shear buckling of orthotropic rectangular plates has been analysed by Seydel,[2] Lekhnitskii,[6] Housner

Fibre Science and Technology 0015-0568/84/$03·00

and Stein[7] and Stein.[8] Shear buckling of unsymmetrical cross ply, square cross ply plates were presented by Whitney.[9] Buckling of boron–aluminum simply supported composite plates with a fiber volume ratio of 0·5, subjected to pure shear load, was examined by Chamis.[10] Design of stiffened laminated plates under biaxial in-plane loads and shear load using Koiter's shear field theory was studied by Dickson *et al.*[11] Shear buckling of any prismatic assembly of anisotropic plates was investigated by Anderson *et al.*[12] and shear buckling of stiffened composite flat plates were considered by Stroud *et al.*[13] However, judging from the available literature, recent review articles[14,15] and a recent book,[16] it is evident that shear buckling and especially postbuckling of unsymmetrically laminated plates has received much less attention. Further, graphical results for both the symmetric and anti-symmetric modes of buckling of anti-symmetric cross ply plates with various aspect ratios have not been presented. In passing, shear buckling of flat and curved, long rectangular laminated plates was examined by Wiswanathan *et al.*[17] and shear buckling of cylindrical and flat panels with unidirectional lay-up and symmetric lay-up was studied by Zhang and Matthews.[18]

The present study was motivated by the physical consideration that for an even-layer anti-symmetric equal-thickness, non-square rectangular cross ply flat plate, a sign change in the amplitude of the geometric imperfection represents a different structure. Such sign dependence disappears if the aspect ratio is one (square plate). This sign-dependent laminated (not necessarily cross ply) plate will possess a non-zero cubic term of the potential energy because of the asymmetric nature of the problem. Thus, at least for extremely small values of the geometric imperfection amplitude, such a structure has unstable initial postbuckling behavior.[19] It was shown in a previous paper by the author[20] that some axially compressed laminated plates which exhibit bending–stretching coupling behavior may possess a non-zero cubic term of the potential energy. Thus, it is of interest to extend the analysis to include the shear loads. Initial postbuckling behavior of anti-symmetric cross ply, rectangular plates under shear loads has not been previously investigated; this is the subject of the present paper. Within the limitations of Koiter's general theory of elastic stability, the present study disagrees with the widely accepted theory that flat plates under in-plane loads (regardless of shape, boundary conditions, etc.) always exhibit stable postbuckling behavior. Thus, the present paper may have significant practical implications for many aircraft and vehicle composite panels under shear loads.

The present study deals with buckling and initial postbuckling of anti-symmetric cross ply, simply supported rectangular flat plates under uniform shear loads. The analysis is based on a Von Kármán-type solution governing equilibrium and compatibility differential equations. These equations are linearized for the buckling problem. The buckling mode shape for the out-of-plane displacement is assumed to be the summation of the double sine series. The compatibility equation is solved exactly by an appropriate choice of the stress function. The Galerkin procedure is used to solve the linearized equilibrium equation approximately. Both the symmetric and anti-symmetric modes of buckling are considered. The resulting system of homogeneous algebraic equations is expressed in terms of the standard eigenvalue problem and the classical buckling load and the associated mode shape are computed using the shifted power method.[21] The asymmetric nature of the initial postbuckling behavior of laminated plates under shear loads is demonstrated by computing the cubic term of the potential energy. However, the asymptotic analysis, especially for small values of the cubic term, is valid only for extremely small values of the imperfection amplitude.

SHEAR BUCKLING OF ANTI-SYMMETRIC CROSS PLY PLATES

The governing nonlinear Von Kármán-type equilibrium and compatibility equations, written in terms of an out-of-plane displacement, W, and a stress function, F, are, respectively:[20,22]

$$\begin{aligned} L_{D^*}(W) + L_{B^*}(F) &= F_{,YY}W_{,XX} + F_{,XX}W_{,YY} - 2F_{,XY}W_{,XY} \\ L_{A^*}(F) - L_{B^*}(W) &= (W_{,XY})^2 - W_{,XX}W_{,YY} \end{aligned} \tag{1}$$

where X and Y are the axial and transverse in-plane coordinates. In order to non-dimensionalize the above differential equations, the following non-dimensional quantities are introduced:

$$\begin{gathered} w = W/h \qquad f = F/(Eh^3) \qquad (x, y) = (X/B, Y/B) \\ (\sigma_x, \sigma_y, \tau) = (f_{,yy}, f_{,xx}, f_{,xy}) = [B^2/(Eh^3)](N_x, N_y, N_{xy}) \end{gathered} \tag{2}$$

where h is the total thickness of the laminated plate, B is the width, and

N'_x, N'_y and N'_{xy} are the membrane stress resultants; the value of reference Young's modulus, E, may be arbitrarily specified. Thus, the non-dimensional equilibrium and compatibility equations, after being linearized with respect to w and f, become:

$$L_{d^*}(w_c) + L_{b^*}(f_c) - \sigma_x w_{c,xx} - \sigma_y w_{c,yy} + 2\tau w_{c,xy} = 0$$
$$L_{a^*}(f_c) - L_{b^*}(w_c) = 0 \qquad (3)$$

For anti-symmetric cross ply plates, the linear differential operators are defined to be:

$$L_{a^*}(\ ) = a^*_{22}(\ )_{,xxxx} + (2a^*_{12} + a^*_{66})(\ )_{,xxyy} + a^*_{11}(\ )_{,yyyy}$$
$$L_{d^*}(\ ) = d^*_{11}(\ )_{,xxxx} + (2)(d^*_{12} + 2d^*_{66})(\ )_{,xxyy} + d^*_{22}(\ )_{,yyyy}$$
$$L_{b^*}(\ ) = b^*_{21}(\ )_{,xxxx} + (b^*_{11} + b^*_{22} - 2b^*_{66})(\ )_{,xxyy} + b^*_{12}(\ )_{,yyyy} \qquad (4)$$

For a cross ply rectangular plate simply supported along all four edges under uniform shear load ($\sigma_x = \sigma_y = 0$), the prebuckling out-of-plane deformation is zero. For the buckling state, the buckling mode can be represented by a summation of the double sine series:

$$w_c(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} c_{mn} \sin(M\pi x) \sin(n\pi y) \qquad (5)$$

where $M = mB/L$ and both m and n are positive integers. It can be seen that the assumed out-of-plane displacement satisfies the zero deflection and zero bending moment conditions at each edge. The buckling compatibility equation can be satisfied exactly by assuming a similar double sine series for the stress function in the form:

$$f_c(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \alpha_{mn} \sin(M\pi x) \sin(n\pi y) \qquad (6)$$

where

$$\alpha_{mn} = [C_{b^*}(M, n)/C_{a^*}(M, n)]c_{mn}$$
$$C_{a^*}(Z_1, Z_2) = a^*_{22}Z_1^4 + (2a^*_{12} + a^*_{66})Z_1^2 Z_2^2 + a^*_{11}Z_2^4$$
$$C_{b^*}(Z_1, Z_2) = b^*_{21}Z_1^4 + (b^*_{11} + b^*_{22} - 2b^*_{66})Z_1^2 Z_2^2 + b^*_{12}Z_2^4 \qquad (7)$$

It may be readily verified that the assumed series for the stress function satisfies the conditions:

$$f_{,xx}\,(\text{at } y=0 \quad \text{or} \quad y=1)=0$$
$$f_{,yy}\,(\text{at } x=0 \quad \text{or} \quad x=L/B)=0 \tag{8}$$

Moreover, the remaining in-plane boundary conditions:

$$(B/L)\int_{x=0}^{L/B} f_{,xy}\,(\text{at } y=0 \quad \text{or} \quad y=1)\,\mathrm{d}x=\tau$$
$$\int_{y=0}^{1} f_{,xy}\,(\text{at } x=0 \quad \text{or} \quad x=L/B)\,\mathrm{d}y=\tau \tag{9}$$

are satisfied on the average.

Substituting the buckling mode $w_c(x, y)$ and $f_c(x, y)$ into the linearized equilibrium equation, one obtains:

$$(\pi^2/\tau)\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_*(M, n)c_{mn}\sin(M\pi x)\sin(n\pi y)\right]$$
$$+\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(2Mn)c_{mn}\cos(M\pi x)\cos(n\pi y)\right]=0 \tag{10}$$

where

$$C_*(Z_1, Z_2)=C_{d*}(Z_1, Z_2)+[C_{b*}(Z_1, Z_2)^2/C_{a*}(Z_1, Z_2)]$$
$$C_{d*}(Z_1, Z_2)=d_{11}^*Z_1^4+(2d_{12}^*+4d_{66}^*)Z_1^2Z_2^2+d_{22}^*Z_2^4 \tag{11}$$

Applying the Galerkin procedure (multiplying both sides by $\sin(P\pi x)\sin(q\pi y)$ and then integrating over the plate area, where $P=pB/L$ and p and q are positive integers), one obtains for given values of p and q,

$$[\pi^4(L/B)C_*(P, q)c_{pq}/(32\tau)]+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{mnpqc_{mn}}{(m^2-p^2)(n^2-q^2)}=0 \tag{12}$$

In the above expression, the summations are taken only for values of the integers m and n which satisfy $m+p=$ odd number and $n+q=$ odd number, and $m \neq p$ and $n \neq q$. The following definite integrals were used to derive the above buckling equation ($C = cB/L$ and $S = sB/L$ and both c and s are positive integers):

$$I(c, s) = (B/L) \int_0^{L/B} \cos(C\pi x) \sin(S\pi x)\, dx$$

$$= -2s/[\pi(c^2 - s^2)] \qquad \text{if } c + s \text{ is odd}; \quad 0 \text{ otherwise}$$

$$H(c, s) = \int_0^1 \cos(c\pi y) \sin(s\pi y)\, dy = I(c, s) \tag{13}$$

The Galerkin procedure yields two uncoupled sets of homogeneous algebraic equations. The symmetric mode of buckling is specified by $p + q =$ even number which implies both m and n are odd or both m and n are even in the summation process in eqn (12). The anti-symmetric mode of buckling is specified by $p + q =$ odd number which implies either m is odd and n is even, or m is even and n is odd. Thus, for the symmetric buckling mode, the values of (p, q) are (1, 1), (1, 3), (2, 2), (3, 1), (1, 5), (2, 4), (3, 3), (4, 2) and (5, 1) for a nine-term series approximation of the out-of-plane deflection. For the anti-symmetric buckling mode, the values of (p, q) are (1, 2), (2, 1), (1, 4), (2, 3), (3, 2), (4, 1), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2) and (6, 1) for a 12-term approximation. For a nine-term approximation of the symmetric mode, the deflection at the plate center is:

$$w(X = L/2, Y = B/2) = c_{11} - c_{13} - c_{31} + (c_{15} + c_{33} + c_{51}) \tag{14}$$

while the deflection at the plate center for the anti-symmetric mode is zero. A larger number of terms is needed for a rectangular plate with a larger aspect ratio, L/B.

The system of homogeneous equations specified by eqn (12) can be written in matrix form, $([Q] - \lambda[B])x = 0$, where $[Q]$ is a symmetric matrix with zeros on the diagonal elements, $[B]$ is a diagonal matrix, $\lambda = 1/\tau$, and x is the solution vector containing c_{pq}. Dividing each equation by the diagonal element in $[B]$, the system of homogeneous equations can be written in the form of a standard eigenvalue problem, $([A] - \lambda[I])x = 0$, where $[I]$ is the identity matrix and $[A]$ is in general no longer a symmetric matrix. The largest eigenvalue in magnitude can be found using the power method. However, the standard power method must be modified in order to accommodate the present case in which the largest eigenvalue occurs in positive and negative pair.[3,9–10] Assuming that λ(actual) + λ(guess) + λ(compute), that is, $\lambda_a = \lambda_g + \lambda_c$, the standard eigenvalue problem can be written in the iterative form:[21]

$$([A] - \lambda_g[I])x_i = \lambda_c x_{i+1} \tag{15}$$

Using a reasonable choice of the initial guess of the eigenvalue, λ_g, and the initial guess of the eigenvector, x_1, the power method converges rapidly with very few iterations. The computed eigenvector is normalized such that the deflection at the plate center specified by eqn (14) is 1.

INITIAL POSTBUCKLING BEHAVIOR

Once the shear buckling load of an anti-symmetric cross ply, rectangular plate is computed, it is of interest to examine the initial postbuckling behavior. Since the purpose of this investigation is to demonstrate that the asymmetry of this type of laminated plate causes imperfection sensitivity, this study is confined to the immediate vicinity of the bifurcation point in the load–deflection curve. Thus, this problem can be examined by computing the cubic term of the potential energy within the context of Koiter's theory of elastic stability.[19]

According to Koiter's theory for single-mode structures, the equilibrium path is specified by:

$$b\xi^3 + a\xi^2 + [1 - (\tau/\tau_c)] = (\tau/\tau_c)\bar{\xi} \tag{16}$$

where a and b are related to the cubic and quartic terms of the potential energy, and ξ and $\bar{\xi}$ are the amplitude of the buckling mode and the geometric imperfection (which is taken to be of the same shape as the buckling mode), respectively. The critical load versus imperfection amplitude equation and the curves for various values of the postbuckling coefficients, a and b, can be found in Ref. 23. Furthermore, a non-zero value of a shows that the structure is imperfection sensitive, at least in an asymptotic sense.

Using Budiansky–Hutchinson's notation[19,24] the a coefficient of a single-mode stability problem is specified by:

$$a = \left(\frac{3(B/L)}{2|Q|}\right) \int_{y=0}^{1} \int_{x=0}^{L/B} [f_{c,yy}(w_{c,x})^2 + f_{c,xx}(w_{c,y})^2 - 2f_{c,xy}w_{c,x}w_{c,y}]\,\mathrm{d}x\,\mathrm{d}y \tag{17}$$

$$Q = (2\tau)(B/L) \int_{y=0}^{1} \int_{x=0}^{L/B} w_{c,x}w_{c,y}\,\mathrm{d}x\,\mathrm{d}y \tag{18}$$

Substituting the assumed double sine series for $w_c(x, y)$ and $f_c(x, y)$ into the above a coefficient expression, one obtains:

$$a = \left(\frac{3\pi^4(B/L)^2}{2|Q|}\right) \sum_{(j,k)} \sum_{(p,q)} \sum_{(m,n)} \{(-k^2mp)I_{scc}H_{sss} - (j^2nq)I_{sss}H_{scc} - (2jkmq)I_{ccs}H_{csc}\}(\alpha_{jk}c_{mn}c_{pq}) \quad (19)$$

$$Q = 2\tau\pi^2(B/L) \sum_{(m,n)} \sum_{(p,q)} \{(mq)I(m, p)H(q, n)\}c_{mn}c_{pq} \quad (20)$$

In the above expressions, the values of (j, k), (p, q) and (m, n) are taken to be $(1, 1), (1, 3), \ldots, (5, 1)$ for the symmetric mode and $(1, 2), (2, 1), \ldots, (6, 1)$ for the anti-symmetric mode; further ($J = jB/L$, $M = mB/L$ and $P = pB/L$):

$$\begin{aligned}
I_{scc} &= (B/L) \int_{x=0}^{L/B} \sin(J\pi x)[\cos(M\pi x)\cos(P\pi x)]\,dx \\
&= (\tfrac{1}{2})[I(m-p, j) + I(m+p, j)] \\
I_{sss} &= (B/L) \int_{x=0}^{L/B} \sin(J\pi x)[\sin(M\pi x)\sin(P\pi x)]\,dx \\
&= (\tfrac{1}{2})[I(m-p, j) - I(m+p, j)] \\
I_{ccs} &= (\tfrac{1}{2})[I(j, m+p) - I(j, m-p)] \\
H_{sss} &= (\tfrac{1}{2})[H(n-q, k) - H(n+q, k)] \\
H_{scc} &= (\tfrac{1}{2})[H(n-q, k) + H(n+q, k)] \\
H_{csc} &= (\tfrac{1}{2})[H(k, n+q) + H(k, n-q)]
\end{aligned} \quad (21)$$

DISCUSSION OF RESULTS

Since it is impossible to present a complete parameter analysis, three types of cross-ply rectangular plates are considered.[16] The material parameters for graphite–epoxy cross ply plates are $E_1/E_2 = 40$, $G_{12}/E_2 = 0{\cdot}5$ and $\nu_{12} = 0{\cdot}25$. Further, the parameters for boron–epoxy cross ply plates are $E_1/E_2 = 10$, $G_{12}/E_2 = 0{\cdot}33$ and $\nu_{12} = 0{\cdot}22$ and the parameters for glass–epoxy cross ply plates are $E_1/E_2 = 3$, $G_{12}/E_2 = 0{\cdot}5$ and $\nu_{12} = 0{\cdot}25$. The quantity E in eqn (2) is chosen to be E_2.

As a check on the analysis, setting $E_1/E_2 = 1$, $G_{12}/E_2 = 1/[2(1+\nu_{12})]$ and $\nu_{12} = 0{\cdot}3$, both the symmetric and anti-symmetric shear buckling loads and the associated buckling modes agree with the results for isotropic, homogeneous rectangular plates under shear loads presented by Stein and Neff[3] and Timoshenko and Gere.[25] It should be noted that:

$$k(\text{Timoshenko}) = 12(1 - \nu_{12}^2)(\tau/\pi) \qquad (22)$$

Figure 1 shows a graph of the non-dimensional shear stress versus the aspect ratio for graphite–epoxy cross ply rectangular plates. Both the symmetric and the anti-symmetric buckling loads are presented for the number of layers, N, being 2, 4 and infinity. The effect of bending–stretching coupling in reducing the shear buckling load is quite pronounced for $N = 2$ and it rapidly becomes less significant for $N = 4$. For an aspect ratio L/B less than 1·7, the symmetric buckling mode is of interest and has one buckle. For an aspect ratio L/B between 1·7 and 2·9, the anti-symmetric buckling mode is of physical significance and has two buckles. Finally, for larger values of the aspect ratio, the symmetric buckling mode is again of interest with three buckles. The symmetric and anti-symmetric shear buckling loads tend to coincide for a plate with a large aspect ratio. It is found that the shear buckling load is independent of the direction of the application of the load in all the example problems presented in this paper.

Figure 2 shows the shear buckling load versus the aspect ratio for boron–epoxy rectangular cross ply plates. The transition values of the aspect ratio are approximately 1·6 and 3·0. A similar graph for glass–epoxy cross ply plates is presented in Fig. 3.

The a coefficient versus the aspect ratio curves for graphite–epoxy, boron–epoxy and glass–epoxy cross ply rectangular plates are shown in Fig. 4. The peak values occur for an aspect ratio of approximately 1·6 for all these types of cross ply plates. The dotted portions of the curves refer to the case when the lowest buckling mode is anti-symmetric; the a coefficient for the anti-symmetric mode of shear buckling is zero. Further, the a coefficient for the orthotropic plate obtained by setting $N =$ infinity is also zero; the $N = 4$ curve rapidly approaches the orthotropic solution. Finally, $a = 0$ for a square plate.

As the values of the a coefficient are found to be very small, the initial postbuckling behavior for realistic finite values of the imperfection

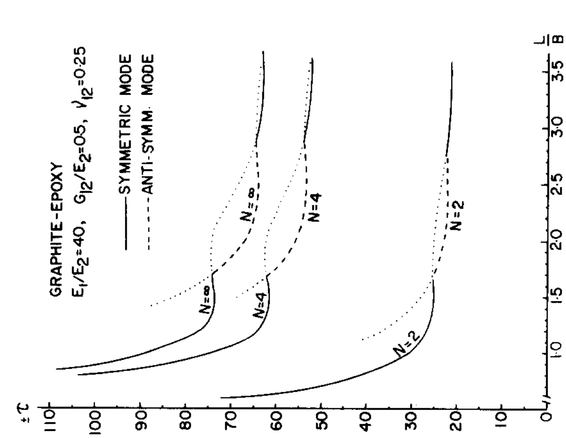

Fig. 1. Shear buckling load versus aspect ratio for anti-symmetric graphite–epoxy simply supported cross ply plates.

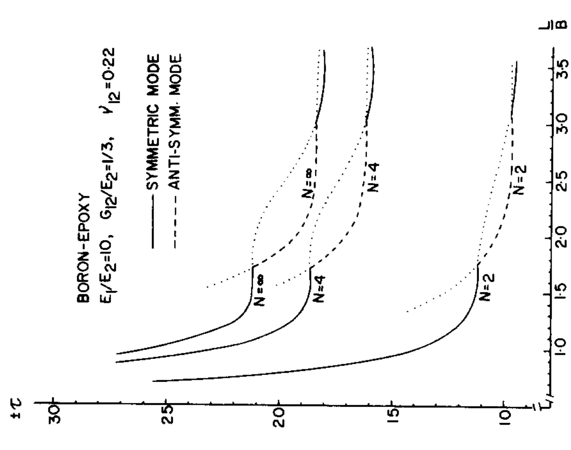

Fig. 2. Shear buckling load versus aspect ratio for anti-symmetric boron–epoxy simply supported cross ply plates.

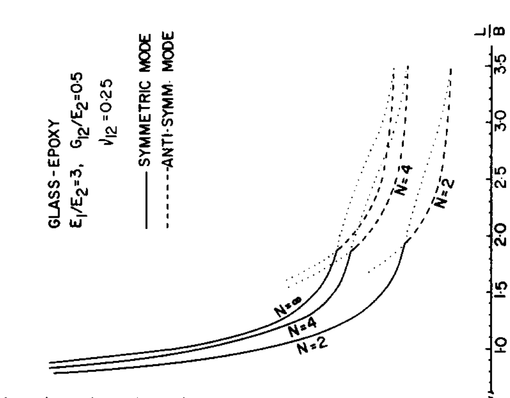

Fig. 3. Shear buckling load versus aspect ratio for anti-symmetric glass–epoxy simply supported cross ply plates.

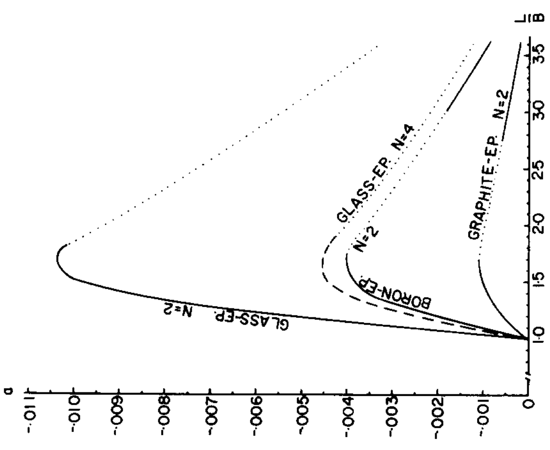

Fig. 4. The postbuckling a coefficient versus aspect ratio for anti-symmetric simply supported rectangular cross ply plates.

amplitude should be affected by the presence of the b coefficient. However, in an asymptotic analysis, at least for extremely small values of the imperfection amplitude, the a coefficient is found to be non-zero, thus substantiating the asymmetry of the initial postbuckling behavior for non-square rectangular, anti-symmetric cross ply plates.

CONCLUDING REMARKS

Buckling of graphite–epoxy, boron–epoxy and glass–epoxy anti-symmetric cross ply plates under uniform shear loads has been investigated. Both the symmetric and anti-symmetric buckling modes were examined along with the effects of the number of layers and of the aspect ratio. The asymmetric nature of the initial postbuckling problem for non-square rectangular cross ply plates is demonstrated by nonzero values of the cubic term of the potential energy.

ACKNOWLEDGEMENTS

This work was partially supported by OSU Office of Research and Graduate Studies. The excellent computer facilities provided by the Instruction and Research Computer Center are also gratefully acknowledged.

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