Composite Structures 3 (1985) 81–95

Asymmetric Postbuckling of Symmetrically Laminated Cross Ply, Short Cylindrical Panels under Compression

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ABSTRACT

Buckling and initial postbuckling behavior of symmetrically laminated, thin cross ply cylindrical panels under axial compression are investigated. The panels are simply supported at all four edges. Closed form solutions are obtained for the buckling loads. The initial asymmetric postbuckling behavior is demonstrated by computing the postbuckling coefficients within the context of Koiter's theory of elastic stability. Parameter studies involving the flatness parameter, the length-to-width ratio, number of layers and Young's moduli ratio are presented for typical cross ply cylindrical panels likely to be encountered in practice.

1 INTRODUCTION

Buckling of fiber-reinforced closed circular cylindrical shells is an important topic due to the increasingly frequent usage of high-strength lightweight composite materials in aerospace and ocean engineering industries. Combined theoretical and experimental investigations on the imperfection sensitivity of axially compressed laminated cylindrical shells were presented by Card,[1] Khot and Venkayya,[2] Tennyson and co-workers,[3,5] Booton[4] and Simitses.[6]

Theoretical studies on the imperfection sensitivity of open, finite-length cylindrical panels under axial compression have also received attention in recent years. Bauld and Satyamurthy[7] developed a comprehensive finite difference Newton–Raphson computer program which is capable of analyzing buckling of geometrically imperfect laminated cylindrical panels. Subsequently, further theoretical results were reported and verified experimentally by Khot and Bauld.[8,9] However, there was no parameter study on the imperfection amplitude, the radius-to-thickness ratio, or the panel length-to-width ratio; no attempt was made in the buckling analyses to determine which structural configuration is the least imperfection sensitive. Further, the influence of the sign of the imperfection amplitude was not presented. Thus, the asymmetric nature of the postbuckling behavior of finite-length cylindrical panels has not been established in the open literature, even for isotropic homogeneous cylindrical panels. The asymmetric postbuckling behavior of symmetrically laminated, geometrically imperfect finite-length cylindrical panels will be analysed in the present paper.

Most of the existing theoretical investigations deal with buckling of perfect (no initial geometric imperfection) laminated cylindrical panels or imperfect isotropic homogeneous cylindrical panels so that the present paper will be invaluable for researchers in composite shell structures, especially in preliminary design which should not involve extensive numerical analysis. For example, buckling of a perfect laminated cylindrical panel subjected to combined in-plane compression and shear load was examined by Viswanathan *et al.*[10] Buckling of perfect, anti-symmetric cross ply graphite-epoxy simply supported cylindrical panels under in-plane compression was studied by Sinha and Rath[11] and Soldatos and Tzivanidis.[12] Buckling and postbuckling of perfect, generally layered cylindrical panels were presented by Zhang and Matthews.[13,14] Imperfection sensitivity of isotropic homogeneous long cylindrical panels under axial compression was examined by Koiter,[15] Stephens[16] and Hui and co-workers.[17,18] Finally, experimental results on axially compressed laminated cylindrical panels were reported by Wilkins,[19] Becker *et al.*[20,21] and Agarwal.[22]

The present buckling and postbuckling analyses are based on a solution of Donnell-type equilibrium and compatibility equations written in terms of an out-of-plane displacement and a stress function. Thus, the results are valid only for sufficiently shallow laminated shell so that the shear deformations may be neglected. These governing differential equations are non-dimensionalized and linearized for the buckling analysis. A closed form eigenvalue expression as a function of the wavelength parameters is found and the classical buckling load is the smallest eigenvalue for all possible discrete number of axial half sine-waves. The asymmetric nature of the initial postbuckling behavior of symmetrically laminated simply supported, finite-length cylindrical panels is studied by computing the cubic terms of the potential energy using Koiter's theory of elastic stability.

The material parameters for the example problems are chosen from typical isotropic homogenous, glass-epoxy and graphite-epoxy I and II materials. Parameter variations involving the flatness parameter, the length-to-width ratio, number of layers and Young's moduli ratio are examined for various symmetrically laminated cross ply cylindrical panels. Particular attention is paid to the possibility of 'beneficial' initial geometric imperfections, depending on the length of the shell and the number of axial half sine-waves; these effects will be of major practical significance in laminated cylindrical panels, likely to be encountered in many structural configurations.

2 BUCKLING AND POSTBUCKLING ANALYSIS

The Donnell-type non-linear equilibrium and compatibility differential equations for a laminated cylindrical panel are,[3-5] respectively:

$$L_{D^*}(W) + L_{B^*}(F) + (1/R)F_{,XX} = F_{,YY}W_{,XX} + F_{,XX}W_{,YY} - 2F_{,XY}W_{,XY} \quad (1)$$

$$L_{A^*}(F) - L_{B^*}(W) - (1/R)W_{,XX} = (W_{,XY})^2 - W_{,XX}W_{YY} \quad (2)$$

where W is the out-of-plane displacement, F is the stress function, R is the shell radius, X and Y are the in-plane axial and circumferential coordinates and $L_{A^*}(\ )$, $L_{B^*}(\ )$ and $L_{D^*}(\ )$ are the linear differential operators. The following non-dimensional quantities are introduced:

$$\begin{aligned} &w = hW, \quad f = F/(Eh^3), \quad (x, y) = (X, Y)/(Rh)^{1/2} \\ &\sigma = |N_x R/(Eh^2)| = -f_{,yy} \\ &a^*_{ij} = EhA^*_{ij}, \quad b^*_{ij} = B^*_{ij}/h, \quad d^*_{ij}/(Eh^3) \end{aligned} \quad (3)$$

where N_x is the membrane stress resultant in the axial direction, h is the total thickness of the laminated shell and E is an arbitrarily specified quantity which has the unit of force per square length. Thus, the non-dimensional linearized equilibrium and compatibility equations for a symmetrically laminated cyindrical panel become, respectively:

$$L_{d^*}(w_c) + f_{c,xx} + \sigma w_{c,xx} = 0 \tag{4}$$

$$L_{a^*}(f_c) - w_{c,xx} = 0 \tag{5}$$

For a symmetrically laminated cross ply structure, the linear operators are defined as[23]

$$L_{a^*}(\ ) = a_{22}^*(\ )_{,xxxx} + (2a_{12}^* + a_{66}^*)(\ )_{,xxyy} + a_{11}^*(\ )_{,yyyy} \tag{6}$$

$$L_{d^*}(\ ) = d_{11}^*(\ )_{,xxxx} + (2)(d_{12}^* + 2d_{66}^*)(\ )_{,xxyy} + d_{22}^*(\ )_{,yyyy} \tag{7}$$

The laminated cylindrical panel is simply supported at all four edges. There is no circumferential stress at the two longitudinal edges. Further, the axial strain at the longitudinal edge is constant. The out-of-plane displacement and the stress function which satisfy the above boundary conditions are

$$\begin{aligned} w_c(x,y) &= \sin(\lambda x/\bar{\theta})\sin(y/\bar{\theta}) \\ f_c(x,y) &= \alpha \sin(\lambda x/\bar{\theta})\sin(y/\bar{\theta}) \end{aligned} \tag{8}$$

where

$$\theta = q_0 d/(2\pi R) = \left(\frac{\sqrt{2c}}{2\pi}\right)\left(\frac{d}{\sqrt{Rh}}\right), \quad \bar{\theta} = \left(\frac{2\theta}{\sqrt{2c}}\right), \quad \lambda = md/L$$

$$c = [3(1-\nu_{12}^2)]^{1/2}, \qquad q_0 = (2cR/h)^{1/2} \tag{9}$$

In the above, θ is the flatness parameter,[15] L is the length of the shell, d is the panel width (defined to be the curved distance between the two longitudinal edges), ν_{12} is Poisson's ratio, m is the number of axial half sine-waves, λ is the ratio of the circumferential buckled wavelength to the axial wavelength and the optimum value of λ is 1 for $L/d = 1$ or ∞.

In the absence of bending–stretching coupling, the pre-buckling out-of-plane deflection is zero. Substituting the buckling mode $w_c(x, y)$ and $f_c(x, y)$ into the linearized compatibility equation, one obtains

$$\alpha = -\lambda^2\bar{\theta}^2/C_{a^*} \tag{10}$$

where

$$C_{a^*} = a_{22}^*\lambda^4 + (2a_{12}^* + a_{66}^*)\lambda^2 + a_{11}^* \tag{11}$$

Substituting the buckling mode w_c and f_c into the linearized and equilibrium equation, the axial compressive load can be written in the form

$$\sigma = \left(\frac{1}{\lambda^2\bar{\theta}^2}\right)[C_{d^*} + (\lambda^4\bar{\theta}^4/C_{d^*})] \tag{12}$$

where

$$C_{d^*} = d^*_{11}\lambda^4 + (2)(d^*_{12} + 2d^*_{66})\lambda^2 + d^*_{22} \tag{13}$$

For a cylindrical panel of given aspect ratio L/d, the integer m which minimizes the eigenvalue expression defined by eqn (12) is the classical buckling load.

The initial postbuckling behavior of a single-mode structure will be examined using Koiter's theory of elastic stability. In terms of Budiansky–Hutchinson's notation,[24] the equilibrium path is specified by

$$b\xi^3 + a\xi^2 + [1 - (\sigma/\sigma_c)]\xi = (\sigma/\sigma_c)\bar{\xi} \tag{14}$$

where ξ and $\bar{\xi}$ are the amplitudes of the buckling mode and the geometric imperfection (taken to be of the same shape as the buckling mode), respectively. In an asymptotic analysis valid for sufficiently small values of the imperfection amplitude, the b coefficient may be neglected in the presence of a non-zero a coefficient so that the critical load of an imperfect system is related to the imperfection amplitude by

$$[1 - (\sigma/\sigma_c)]^2 = -4a\bar{\xi}(\sigma/\sigma_c) \tag{15}$$

If the assumption that the b coefficient may be neglected is valid and if the a coefficient turns out to be a positive quantity, then the above structure will be able to carry loads above the classical buckling load for positive values of the imperfection amplitude.

The a coefficient can be expressed in terms of the stress function and the out-of-plane deflection in the form[24]

$$a = \left(\frac{3}{2|Q|}\right)\int_{y=0}^{\pi\bar{\theta}}\int_{x=0}^{\pi\bar{\theta}L/d} f_{c,xx}(w_{c,y})^2 + f_{c,yy}(w_{c,x})^2 - 2f_{c,xy}(w_{c,x})(w_{c,y})\,\mathrm{d}x\,\mathrm{d}y \tag{16}$$

$$Q = \sigma_c\int_{y=0}^{\pi\bar{\theta}}\int_{x=0}^{\pi\bar{\theta}L/d}(w_{c,x})^2\mathrm{d}x\,\mathrm{d}y \tag{17}$$

Substituting the buckling mode $w_c(x, y)$ and $f_c(x, y)$ and the classical buckling load σ_c into the above a-coefficient expression and performing the axial and circumferential integrations, one obtains

$$Q = (\sigma_c \pi^2 \lambda^2/4)(L/d) \tag{18}$$

$$a = \begin{cases} \dfrac{-16\alpha}{\sigma_c \bar{\theta}^2 \pi^2 \lambda (L/d)} & \text{for odd } m \text{ values} \\ 0 & \text{for even } m \text{ values} \end{cases} \tag{19}$$

In deriving the above a-coefficient expression, the following integration formulae were employed:

$$\int_{x=0}^{\pi\bar{\theta}L/d} \sin^3(\lambda x/\bar{\theta})\,dx = 4\bar{\theta}/(3\lambda) \quad \text{if } m \text{ is odd; 0 otherwise}$$

$$\int_{x=0}^{\pi\bar{\theta}L/d} \sin(\lambda x/\bar{\theta})\cos^2(\lambda x/\bar{\theta})\,dx = 2\bar{\theta}/(3\lambda) \quad \text{if } m \text{ is odd; 0 otherwise}$$

$$\int_{y=0}^{\pi\bar{\theta}} \sin^3(y/\bar{\theta})\,dy = 4\bar{\theta}/3$$

$$\int_{y=0}^{\pi\bar{\theta}} \sin(y/\bar{\theta})\cos^2(y/\bar{\theta})\,dy = 2\bar{\theta}/3 \tag{20}$$

In the important special case of an isotropic homogeneous cylindrical panel under axial compression, the governing linearized equilibrium and compatibility equation can be written, using the same non-dimensionalization scheme, respectively, in the forms:

$$[1/(4c^2)](w_{c,xxxx} + w_{c,yyyy} + 2w_{c,xxyy}) + f_{c,xx} + \sigma w_{c,xx} = 0 \tag{21}$$

$$(f_{c,xxxx} + f_{c,yyyy} + 2f_{c,xxyy}) - w_{c,xx} = 0 \tag{22}$$

A direct comparison with eqns (4) and (5) yields

$$\begin{aligned} d^*_{11} &= (d^*_{12} + 2d^*_{66}) = d^*_{22} = 1/(4c^2) \\ a^*_{22} &= 1, \qquad 2a^*_{12} + a^*_{66} = 2, \qquad a^*_{11} = 1 \end{aligned} \tag{23}$$

Thus, the values of the coefficient of the stress function, the wavelength parameter Ω, the axial buckling load and the a coefficient for isotropic cylindrical panels are

$$\alpha = (-\bar{\theta}^2/\Omega) = -2\theta^2/(c\Omega) \tag{24}$$

$$\Omega = (\lambda^2 + 1)^2/\lambda^2 \tag{25}$$

$$\sigma_c = [\Omega + (16\theta^4/\Omega)]/(8c\theta^2) \tag{26}$$

$$a = \left(\frac{16}{\sigma_c \pi^2 (L/d)}\right)\left(\frac{\lambda}{\lambda^4 + 2\lambda^2 + 1}\right) \tag{27}$$

3 RESULTS AND DISCUSSION

Buckling of symmetrically laminated (odd number of equal-thickness layers) cross ply, axially compressed cylindrical panels, simply supported at all four edges, is studied. The stacking sequences for three-layered and five-layered panels are (0°, 90°, 0°) and (0°, 90°, 0°, 90°, 0°), respectively. Each layer is orthotropic and five types of material parameters are considered ($\nu_{21} = \nu_{12}E_2/E_1$):

(i) isotropic homogeneous, $E_1/E_2 = 1$, $G_{12}/E_2 = 1/[2(1+\nu_{12})]$, $\nu_{12} = 0{\cdot}3$;
(ii) glass–epoxy, $E_1/E_2 = 3{\cdot}0$, $G_{12}/E_2 = 0{\cdot}5$, $\nu_{12} = 0{\cdot}25$;
(iii) boron–epoxy, $E_1/E_2 = 10{\cdot}0$, $G_{12}/E_2 = 0{\cdot}3333\,33$, $\nu_{12} = 0{\cdot}22$;
(iv) graphite–epoxy I, $E_1/E_2 = 15{\cdot}769\,2$, $G_{12}/E_2 = 0{\cdot}576\,923$, $\nu_{12} = 0{\cdot}335$;
(v) graphite–epoxy II, $E_1/E_2 = 40{\cdot}0$, $G_{12}/E_2 = 0{\cdot}5$, $\nu_{12} = 0{\cdot}25$.

Figure 1 shows the equilibrium paths of a single-mode asymmetric structural system. In the figure, σ/σ_c is the applied load normalized with the buckling load, ξ and $\bar{\xi}$ are the amplitudes of the buckling mode and geometric imperfection, respectively. The value of the a coefficient is 0·669 64 and this corresponds to the postbuckling behavior of an isotropic homogeneous, compressed cylindrical panel simply supported at all four edges with $\theta = 1{\cdot}0$, $L/d = 1{\cdot}0$ and Poisson's ratio = 0·3. These curves are plotted by assuming that the quartic term of the potential energy represented by the b coefficient may be neglected in the presence of a non-zero cubic term represented by the a coefficient. It can be seen that the equilibrium paths for positive values of the amplitude of the geometric imperfection (deviation from the perfect cylindrical shape by bulging outward as viewed from the interior of the cylindrical shell) will not intersect the stability boundary and thus the structure will be able to carry the applied load above the classical buckling load of the perfect system. On the other hand, the equilibrium paths for negative values of the imperfection amplitude (bulging inwards) will intersect the stability boundary and, thus, the structure will be imperfection sensitive and it will not be able to carry the load above the classical buckling load. Typical values of the equilibrium paths for imperfection amplitude, ±0·025, ±0·05, ±0·1 and ±0·2, are plotted. Finally, due to the asymptotic nature of Koiter's theory of elastic stability, the above behavior is valid only for sufficiently small values of the imperfection amplitude; otherwise, it should at best be viewed as a qualitative representation.

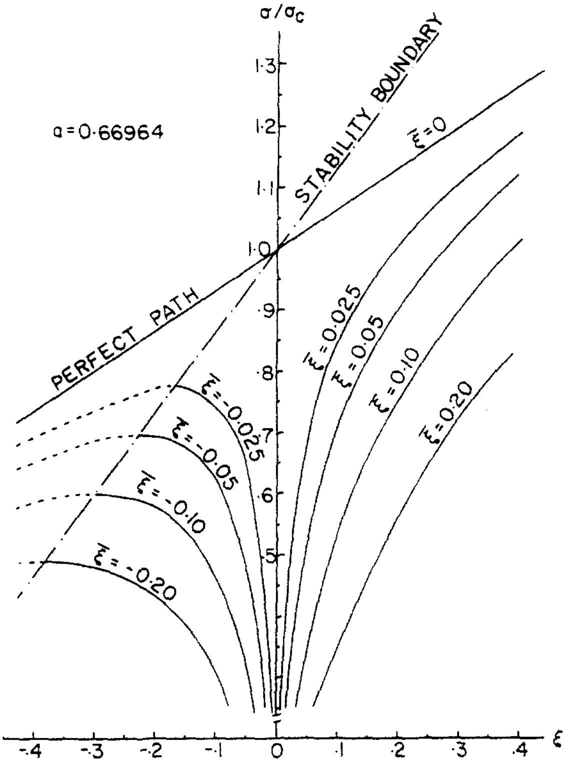

Fig. 1. Equilibrium paths of a single-mode asymmetric structural system (a = 0·669 64).

Figure 2a shows a graph of the applied axial buckling load normalized with respect to the reference buckling load length-to-width versus the flatness parameter θ for the ratio L/d = 1·0 and an infinite number of layers (orthotropic). The reference buckling loads ($\sigma_c^{ref} = \sigma_c(\theta = L/d = 1{\cdot}0,\ N = \infty)$) for isotropic homogeneous, glass-epoxy, boron-epoxy, graphite-epoxy I and graphite-epoxy II simply supported cylindrical panels are 0·605 227, 0·876 365, 1·256 72, 1·976 18 and 3·613 10 respectively. The range of θ values is chosen to be less than 1·2 in order for the present single-mode analysis to be valid and these are the values which correspond to many practical closely-spaced axially-stiffened, closed laminated cylindrical shells. It can be seen that the buckling load is higher for smaller values of the flatness parameter. A small value of the flatness parameter indicates that the width of the panel

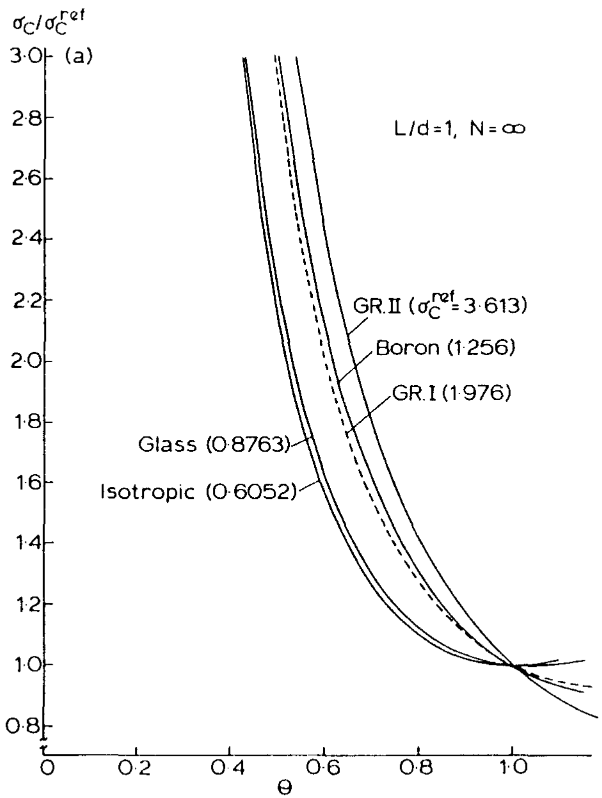

Fig. 2a. Normalized buckling load versus flatness parameter for orthotropic cross ply cylindrical panels ($L/d = 1$).

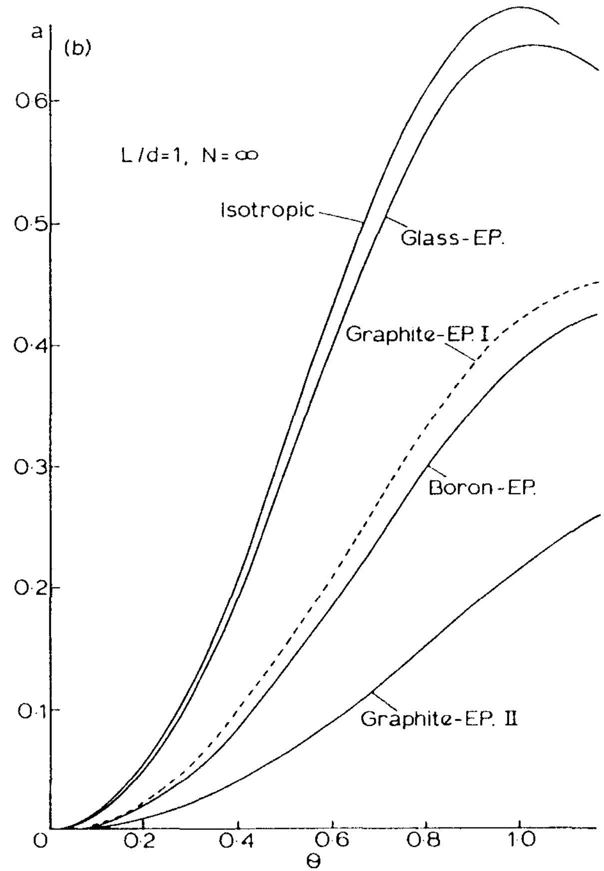

Fig. 2b. The a coefficient versus flatness parameter for orthotropic cross ply cylindrical panels ($L/d = 1$).

d is small or the product Rh is large ($\theta = 0$ for a flat plate). The flatness parameter is independent of the length of the cylindrical panel. The optimum number of axial half sine-waves which minimizes the buckling load is always one for an orthotropic cylindrical panel with $L/d = 1{\cdot}0$. In comparing these curves, it should be noted that the reference buckling loads are different for each type of material.

The corresponding graph of the postbuckling a coefficient versus the flatness parameter is depicted in Fig. 2b for $L/d = 1$ and $N = \infty$. Since the a coefficients are found to be always positive, the buckling load of the structure is sensitive only to the presence of geometric imperfection which bulges inwards ($\xi < 0$). Within the range of values of the flatness parameter being considered, it appears in general that square ($L/d = 1$), orthotropic ($N = \infty$) simply supported cylindrical panels are less imperfection sensitive for large values of Young's moduli ratio ($E_1/E_2 = 40$). The magnitude of the a coefficient is largest for the cylindrical panel made of isotropic homogeneous material. The a coefficient is, identically, zero in the limiting case of flat, laminated or isotropic plates. Typical equilibrium paths for $a = 0{\cdot}669\,640$ are shown in Fig. 1.

A graph of the normalized axial buckling load versus the flatness parameter for symmetrically laminated graphite-epoxy I simply supported cylindrical panels for the length-to-width ratio $L/d = 0{\cdot}7$, $1{\cdot}0$ and $1{\cdot}4$ and number of layers $N = 3, 5$ and ∞ is presented in Fig. 3a. For a square-shaped ($L/d = 1$) laminated cylindrical panel, the buckling load is nearly independent of the number of layers (too small to be plotted clearly). For a shorter panel ($L/d = 0{\cdot}7$) with θ held fixed, the buckling load is lowered as the number of layers is increased ($N = 3, 5, \ldots$). On the other hand, for a laminated cylindrical panel with larger length-to-width ratio ($L/d = 1{\cdot}4$), the buckling load is increased as the number of layers is increased ($N = 3, 5, \ldots$).

The reason for the above behavior can be explained by a graph of the normalized buckling load versus the length-to-width ratio in Fig. 4a for flatness parameter $\theta = 1{\cdot}0$. Simultaneous buckling of the laminated cylindrical panel which involves the $m = 1$ and $m = 2$ modes occurs at $L/d = 1{\cdot}41$, $1{\cdot}9$ and $2{\cdot}6$ for $N = \infty, 5$ and 3, respectively. The larger value of the transitional L/d for the three-layered laminated cylindrical panel is due to the fact that there are two layers with 0° fiber and only one layer with 90° fiber. Thus, the panel is stiffer against bending in the x-direction than in the y-direction so that $m = 1$ will yield the smallest buckling load for a larger range of the length-to-width ratio.

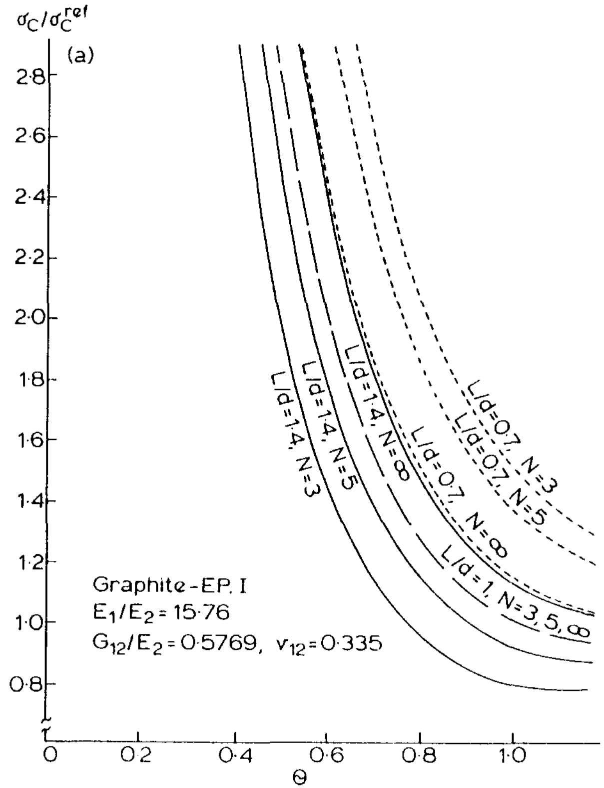

Fig. 3a. Normalized buckling load versus flatness parameter for symmetrically laminated graphite-epoxy I cross ply cylindrical panels (L/d = 0·7, 1·0 and 1·4).

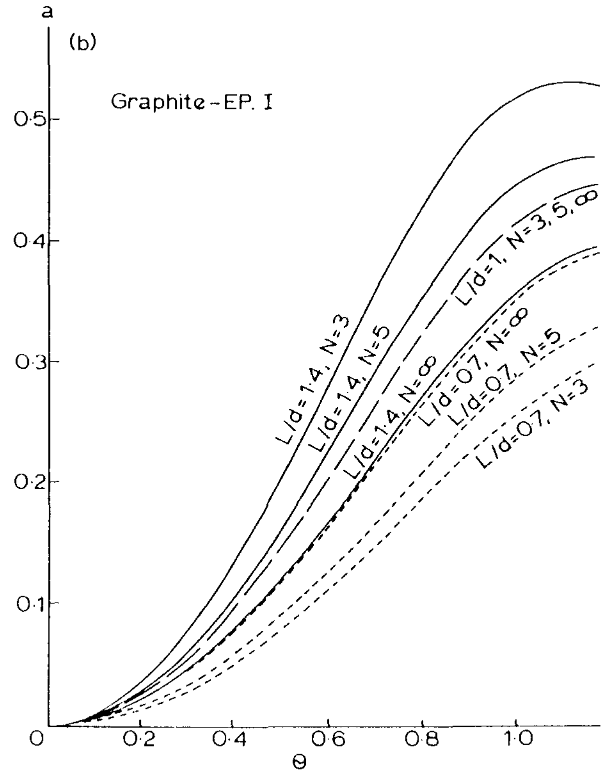

Fig. 3b. The *a* coefficient versus flatness parameter for symmetrically laminated graphite-epoxy I cross ply cylindrical panels (L/d = 0·7, 1·0 and 1·4).

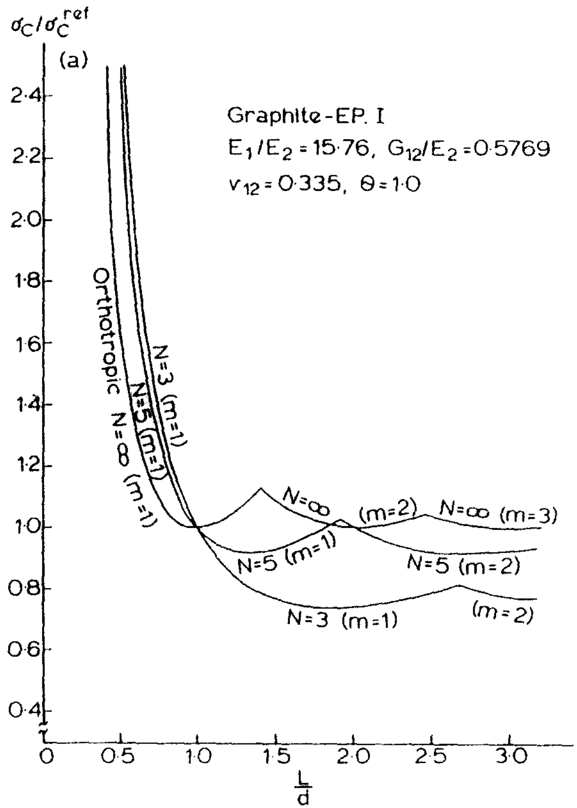

Fig. 4a. Normalized buckling load versus the length-to-width ratio for symmetrically laminated graphite-epoxy I cross ply cylindrical panels ($\theta = 1.0$).

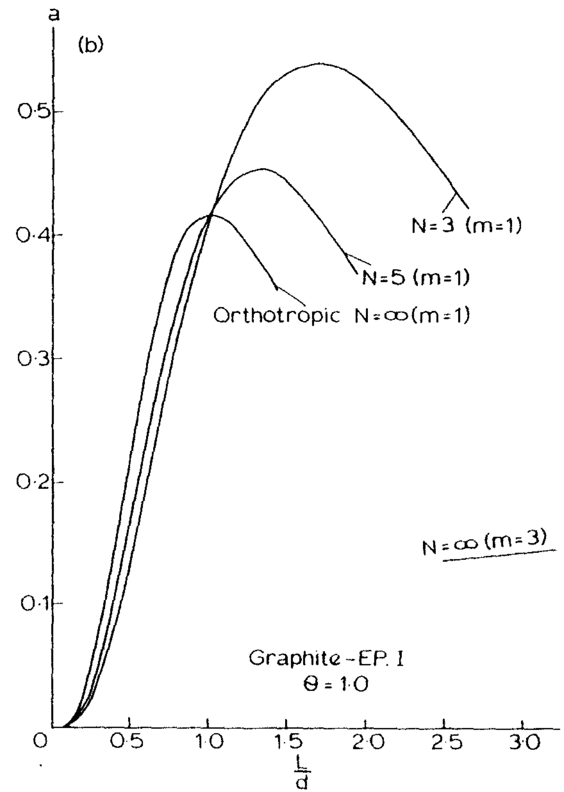

Fig. 4b. The a coefficient versus the length-to-width ratio for symmetrically laminated graphite-epoxy I cross ply cylindrical panels ($\theta = 1.0$).

The corresponding a coefficient versus the flatness parameter curves for graphite-epoxy I laminated simply supported cylindrical panels is plotted in Fig. 3b. For the length-to-width ratio $L/d = 1$, the a coefficient is again nearly independent of the number of layers. For a shorter panel with fixed flatness parameter and $L/d = 0{\cdot}7$, the a coefficient increases with increasing number of layers ($N = 3, 5, \ldots, \infty$). However, for a longer cylindrical panel, the a coefficient decreases with increasing number of layers. All curves pass through the origin in the case of a laminated flat plate ($\theta = 0$).

Finally, a graph of the a coefficient versus the length-to-width ratio L/d for a graphite-epoxy I symmetrically laminated cross ply cylindrical panel, simply supported at all four edges, is presented in Fig. 4b. It can be seen that the three-layered laminated panel has a higher peak value of the a coefficient, indicating that the structure is more imperfection sensitive than the $N = 5$ or the orthotropic panels for aspect ratio $L/d > 1$. The reverse is true for a shorter panel with $L/d < 1$. It should be cautioned that the a coefficient is identically zero for a buckling mode with even number axial half-waves.

As a check on the analysis, the buckling load and the buckling mode agreee with the results presented by Koiter[15] and Stephens[16] for the special case of simply supported, long isotropic homogeneous cylindrical panels. Since the present single-mode analysis is valid only for small values of the flatness parameter, it is not possible to compare the buckling solution with existing results.

4 CONCLUSIONS

Buckling and asymmetric postbuckling of symmetrically laminated cross ply simply supported cylindrical panels under axial compression have been studied. The results for the parameter variation which involves the flatness parameter, the length-to-width ratio, number of layers and five material parameters have been presented. It appears that the cylindrical panel may be designed to carry loads above the classical buckling load by introducing the geometric imperfection in the shape which initially bulges outwards. It may be conjectured that the present results are qualitatively valid for a wide variety of finite length laminated cylindrical panels such as anti-symmetrically laminated angle[25] and cross ply[26] cylindrical panels and for other boundary loading conditions.[27] Finally, it should be noted that the present analysis is valid only for sufficiently small values of the imperfection amplitudes due to the asymptotic nature of Koiter's theory of elastic stability.

ACKNOWLEDGEMENTS

The author is grateful for a research grant awarded by the OSU Office of Research and Graduate Studies. The computational facilities provided by the instruction and research computer center are also acknowledged with thanks.

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