Effects of Axial Imperfections on Vibrations of Anti-Symmetric Cross-Ply, Oval Cylindrical Shells

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This paper examines the effects of axial geometric imperfections on the fundamental vibration frequencies of cross-ply simply-supported oval cylindrical shells. It is found that the presence of such imperfection with small amplitudes may significantly raise or lower the fundamental frequencies, depending on the wave numbers of the imperfection and vibration mode. The effects of oval eccentricity, bending-stretching coupling of the material, the reduced-Batdorf parameter and Young's moduli ratio are examined. It appears that the present problem has not been examined, even in the simplified case of oval cylindrical shells made of isotropic-homogeneous material.

1 Introduction

The effects of geometric imperfections on the vibration frequencies of isotropic-homogeneous shells have been thoroughly analyzed by a number of researchers. For example, the influence of geometric imperfections on circular cylindrical shells were examined by Rosen and Singer (1974, 1976), Koval'chuk and Krasnopol'skaya (1980), Singer and Prucz (1982), and Watawala and Nash (1983). Similar analyses on open, simply-supported, cylindrical panels within plane contraints (Hui 1984), shallow spherical shells segments (Hui and Leissa, 1983, and Hui, 1983) have also been reported. However, all of these studies have been concerned with isotropic-homogeneous materials.

The influence of geometric imperfections on the fundamental frequencies of anti-symmetrically angle or cross-ply rectangular plates were examined by the author (Hui 1985a, b). Further analyses of anti-symmetric angle-ply simply-supported cylindrical panels were performed by Du and Hui (1984). More complicating effects involving elastically restrained edges and elastic foundation were also studied by Bhattacharya (1984); however, his analysis was restricted to symmetricaly laminated cylindrical panels. The effects of general imperfections (that is, not necessarily the same shape as the vibration mode) on vibrations of laminated circular cylindrical shells using the finite element method were studied by Yang and Kapania (1985) and Kapania and Yang (1986). Due to storage and maneuverability requirements of aircraft fuselages, submarine hulls, and cylindrical tanks, it is of interest to examine the effects of noncircularity on vibrations of cylindrical shells. This problem is also of interest to the basic applied mechanics community since it appears that the effects of imperfections on even the isotropic-homogeneous oval shells has not been previously reported. In passing, vibrations of "perfect isotropic-homogeneous" noncircular cylindrical shells were examined by Malkina (1967), Sewell et al. (1971), Culberson and Boyd (1971), Chen and Kempner (1978), Koumousis and Armenakas (1983), and Suzuki and Leissa (1985), among others. Vibrations of "perfect-laminated" oval cylindrical shells were examined by Sathyamoorthy and Pandalai (1970), Soldatos and Tzivanidis (1982), and Soldatos (1983, 1984, 1985).

The present analysis is based on a solution of Donnell-type linearized equilibrium and compatibility equations of laminated oval cylindrical shell. The shell is assumed to be simply supported with no in-plane shear load and no axial load at both ends. The compatibility equation is satisfied exactly by an appropriate choice of the stress function and the equilibrium equation is satisfied approximately by a Galerkin procedure. Thus, the computed frequencies are "upper-bounds." An iterative inverse power method is used to solve for the smallest eigenvalue. The fundamental frequencies are obtained by solving for the smallest eigenvalue for all possible axial wave number and circumferential boundary conditions.

In order to simplify the analysis, the geometric imperfection of the oval cylinders are assumed to be axisymmetric (that is, no circumferential waves). The number of waves in the axial direction of the geometric imperfections need not be the same as that for the vibration mode. Thus, two possibilities arise: if the effects of axial imperfection are found to affect significantly the fundamental frequencies, then one cannot safely neglect the effects of "general" imperfections which occur in practice. On the other hand, if these effects are not important, then no conclusion can be drawn about the effects of "general" imperfections.

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2 Vibrations of Oval Cylindrical Shells

The vibration analysis is based on a solution of Donnell-type equilibrium and compatibility equations for "generally" laminated oval cylindrical shells, written in terms of an out-of-plane deflection W, the stress function F, incorporating the possibility of an axisymmetric (that is, axial) geometric imperfection W_o. They are, respectively,

$$\rho W_{,\bar{t}\bar{t}} + L_{D^*}(W) + L_{B^*}(F) + (1/R(Y))(F_{,XX}) = F_{,YY}(W+W_o)_{,XX} + F_{,XX}W_{,YY} - 2F_{,XY}W_{,XY} \quad (1)$$

$$L_{A^*}(F) = L_{B^*}(W) + (W_{,XY})^2 - (W+W_o)_{,XX}W_{,YY} + [1/R(Y)]W_{,XX} \quad (2)$$

In the above equations, X and Y are the axial and circumferential coordinates, ρ is the mass of the shell per unit surface area, $\bar{t}$ is time, and $R(Y)$ is the variable radius of the oval cylindrical shell. The linear differential operators $L_{A^*}(\ )$, $L_{B^*}(\ )$ and $L_{D^*}(\ )$ are defined by Tennyson et al. (1971) and they are defined later in nondimensional form in equations (9). Further, each layer is orthotropic and (A_{ij}, B_{ij}, D_{ij}) and $(A^*_{ij}, B^*_{ij}, D^*_{ij})$ are defined to be, (Ashton et al. 1969),

$$(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^{n} (\bar{Q}_{ij})_k (h_k - h_{k-1}, h_k^2 - h_{k-1}^2, h_k^3 - h_{k-1}^3) \quad (3a)$$

$$[A^*_{ij}] = [A_{ij}]^{-1},\ [B^*_{ij}] = -[A_{ij}]^{-1}[B_{ij}],\quad [D^*_{ij}] = [D_{ij}] - [B_{ij}][A_{ij}]^{-1}[B_{ij}] \quad (3b)$$

Note that for the kth layer, the principal stresses are related to the principal strains such that $[\sigma_1, \sigma_2, \tau_{12}]^T = [Q_{ij}][\epsilon_1, \epsilon_2, \gamma_{12}]^T$; where $Q_{16} = 0$, $Q_{26} = 0$ for the above mentioned orthotropic lamina. For an arbitrary set of axes in the kth layer, $[\sigma_x, \sigma_y, \tau_{xy}]^T = [\bar{Q}_{ij}][\epsilon_x, \epsilon_y, \gamma_{xy}]^T$ where $\bar{Q}_{11}, \bar{Q}_{22}, \bar{Q}_{12}, \bar{Q}_{66}$ are even functions of the fiber angle while $\bar{Q}_{16}, \bar{Q}_{26}$ are odd functions of the fiber angle.

The following nondimensional quantities are introduced,

$$(a^*_{ij}, b^*_{ij}, d^*_{ij}) = (E_2hA^*_{ij}, B^*_{ij}/h, D^*_{ij}/(E_2h^3))$$
$$(x, y) = (X, Y)/(R_oh)^{1/2}$$
$$(w, w_o) = (W/h, W_o/h), f = F/(E_2h^3)$$
$$t = \omega_r\bar{t},\ (\omega_r)^2 = E_2h/(\rho R_o^2) \quad (4)$$

where ω_r is the reference frequency, h is the total thickness of the laminated shell, E_2 is Young's modulus of a particular layer in the transverse direction, and the average radius of the oval cylindrical shell R_o can be obtained from,

$$\int_{\theta=0}^{2\pi} R(Y)d\theta = 2\pi R_o \quad (5)$$

Thus, the nondimensional linearized Donnell-type equilibrium and compatibility equations for laminated oval cylindrical shell becomes,

$$w_{,tt} + L_{d^*}(w) + L_{b^*}(f) + [1/r(y)]f_{,xx} = w_{o,xx}f_{,yy} \quad (6)$$

$$L_{a^*}(f) = L_{b^*}(w) + [1/r(y)]w_{,xx} - w_{o,xx}w_{,yy} \quad (7)$$

where ϵ is the oval eccentricity parameter and $r(y) = R(y)/R_o$ can be obtained from (see Chen and Kemper, 1971, and Soldatos, 1984),

$$1/r(y) = 1 + \epsilon\cos(2Y/R_o) \quad (8)$$

For an antisymmetric cross-ply laminate, the nondimensional linear operators are defined to be (Hui, 1986),

$$L_{a^*}(\ ) = a^*_{22}(\ )_{,xxxx} + (2a^*_{12} + a^*_{66})(\ )_{,xxyy} + a^*_{11}(\ )_{,yyyy}$$
$$L_{b^*}(\ ) = b^*_{21}(\ )_{,xxxx} + (b^*_{11} + b^*_{22} - 2b^*_{66})(\ )_{,xxyy} + b^*_{12}(\ )_{,yyyy}$$
$$L_{d^*}(\ ) = d^*_{11}(\ )_{,xxxx} + (2)(d^*_{12} + 2d^*_{66})(\ )_{,xxyy} + d^*_{22}(\ )_{,yyyy} \quad (9)$$

The axial geometric imperfection and the vibration mode are assumed to be of the form,

$$w_o(x) = \mu\sin(Jx) \quad (10a)$$
$$w(x,y) = w(y)\sin(Mx) \quad (10b)$$

In the above, μ is the amplitude of the imperfection and the wave number J and M are defined in terms of the reduced-Batdorf parameter Z_H by,

$$(J, M) = (j\pi/Z_H, m\pi/Z_H),\ Z_H = L/(R_oh)^{1/2} \quad (11)$$

where the integers j and m are number of axial half-waves.

It can be seen from the compatibility equation that the analysis would be much simpler if the strectching-twisting coupling coefficients vanish so that from now on, $a^*_{16} = 0$, $a^*_{26} = 0$. This restriction means that for each layer of a plus-positive fiber angle, there exists another layer of the same orthotropic properties and thickness with a negative fiber angle since $\bar{Q}_{16}$ and $\bar{Q}_{26}$ are odd function of the fiber angle. Thus, the stress function which satisfies the linearized compatibility equation exactly is,

$$f(x,y,t) = f_A(y)\sin(Mx) + f_B(y)\cos(Mx) + f_C(y)\cos[(J-M)x] + f_D(y)\cos[(J+M)x] + (c_x)(x^2/2) + (c_y)(y^2/2) \quad (12)$$

For ordinary differential equations are obtained from this compatibility equation by collecting terms involving $\sin(Mx)$, $\cos(Mx)$, $\cos[(J-M)x]$, and $\cos[(J+M)x]$, respectively:

$$a^*_{11}f_A(y)_{,yyyy} + (-M^2)(2a^*_{12} + a^*_{66})f_A(y)_{,yy} + M^4a^*_{22}f_A(y) = b^*_{12}w(y)_{,yyyy} - M^2(b^*_{11} + b^*_{22} - 2b^*_{66})w(y)_{,yy} + [b^*_{21}M^4 - (M^2/r(y))]w(y) \quad (13)$$

$$a^*_{11}f_B(y)_{,yyyy} + (-M^2)(2a^*_{12} + a^*_{66})f_B(y)_{,yy} + M^4a^*_{22}f_B(y) = (2b^*_{26} - b^*_{61})(-M^3)w(y)_{,y} + (2b^*_{16} - b^*_{62})(M)w(y)_{,yyy} \quad (14)$$

$$a^*_{11}f_C(y)_{,yyyy} - (J-M)^2(2a^*_{12} + a^*_{66})f_C(y)_{,yy} + (J-M)^4a^*_{22}f_C(y) = (\mu J^2/2)w(y)_{,yy} \quad (15)$$

$$a^*_{11}f_D(y)_{,yyyy} - (J+M)^2(2a^*_{12} + a^*_{66})f_D(y)_{,yy} + (J+M)^4a^*_{22}f_D(y) = (-\mu J^2/2)w(y)_{,yy} \quad (16)$$

The remaining ordinary differential equation can be obtained from the linearized equilibrium equation using a Galerkin procedure (that is, multiplying both sides by $\sin(Mx)$ and integrating from $x=0$ to $x=Z_H$),

$$\{-(\omega/\omega_r)^2w(y) + d^*_{22}w(y)_{,yyyy} - 2M^2(d^*_{12} + 2d^*_{66})w(y)_{,yy} + d^*_{11}M^4w(y) + b^*_{12}f_A(y)_{,yyyy} + (b^*_{11} + b^*_{22} - 2b^*_{66})(-M^2)f_A(y)_{,yy} + [b^*_{21}M^4 - (M^2/r(y))]f_A(y) - M(2b^*_{16} - b^*_{62})f_B(y)_{,yyy} + (M^3)(2b^*_{26} - b^*_{61})f_B(y)_{,y}\}(1/2) + \{b^*_{12}f_C(y)_{,yyyy} - (b^*_{11} + b^*_{22} - 2b^*_{66})(J-M)^2f_C(y)_{,yy} + [(J-M)^4b^*_{21} - (M^2/r(y))]f_C(y)\}I_1 + \{b^*_{12}f_D(y)_{,yyyy} - (b^*_{11} + b^*_{22} - 2b^*_{66})(J+M)^2f_D(y)_{,yy} + [(J+M)^4b^*_{21} - (M^2/r(y))]f_D(y)\}I_2 = (I_3)(-J^2\mu)f_A(y)_{,yy} \quad (17)$$

where the definite integrals I_1, I_2, and I_3 are defined to be,

$$I_1 = (1/Z_H)\int_{x=0}^{Z_H}\cos[(J-M)x]\sin(Mx)\,dx = [1/(2Z_H)]\{(-2/J) + [2/(J-2M)]\} \text{ if } j \text{ is odd; } 0 \text{ otherwise} \quad (18)$$

$$I_2 = (1/Z_H)\int_{x=0}^{Z_H} \cos[(J+M)x] \sin(Mx)\, dx$$
$$= [1/(2Z_H)]\{[-2/(J+2M)] + (2/J)\} \text{ if } j \text{ is odd;}$$
$$0 \text{ otherwise} \quad (19)$$

$$I_3 = (1/Z_H)\int_{x=0}^{Z_H} \sin[(Jx) \quad \sin^2(Mx)\, dx$$
$$= [1/(2Z_H)]\{(-2/J) + [1/(J+2M)] + [1/(J-2M)]\}$$
$$\text{if } j \text{ is odd;}$$
$$0 \text{ otherwise} \quad (20)$$

The above five coupled ordinary homogeneous differential equations with five variables $f_A(y), f_B(y), f_C(y), f_D(y)$ and $w(y)$ are discretized using a central finite difference scheme. Due to symmetry considerations, the deflection and the stress function of the oval cylindrical shell must be either symmetric or antisymmetric at the semi-major $(Y=0)$ and the semi-minor $(Y=\pi R_o/2)$ axes. Thus, only one-quarter of the shell needs to be discretized. Further, the deflection mode can be identified as one of the following four groups: *SS* (symmetric at both axes), *AA* (antisymmetric at both axes), *SA* (symmetric at the semi-major axis and antisymmetric at the semi-minor axis), and *AS* (antisymmetric at the semi-major axis and symmetric at the semi-minor axis). Thus, the boundary conditions at either the semi-major axis $(Y=0)$ or the semi-minor axis $(Y=\pi R_o/2)$ for the symmetric mode are (Hutchinson 1968),

$$w_{,y}=0,\ w_{,yyy}=0,$$
$$f_{A,y}=0, f_{B,y}=0, f_{C,y}=0, f_{D,y}=0,$$
$$f_{A,yyy}=0, f_{B,yyy}=0, f_{C,yyy}=0, f_{D,yyy}=0 \quad (21)$$

For the antisymmetric mode, the boundary conditions are,

$$w=0,\ w_{,yy}=0,$$
$$f_A=0, f_B=0, f_C=0, f_D=0,$$
$$f_{A,yy}=0, f_{B,yy}=0, f_{C,yy}=0, f_{D,yy}=0 \quad (22)$$

The out-of-plane simply-supported boundary conditions are satisfied exactly by the choice of the mode shape. The zero in-plane shear stress condition is also satisfied identically at the two ends of the cylindrical shell. The zero axial stress and zero lateral pressure conditions are also satisfied approximately by setting $c_x=0$, $c_y=0$. The total number of circumferential full-waves is defined to be $\bar{n}$ and is equal to half the number of nodal points of the deflection mode in a complete circumferential circuit. The symbol n is not used in this paper ($\bar{n}$ refers to the finite difference mode shape and n refers to the trigonometric modes used by Soldatos, 1984). Due to singlevaluedness and continuity of the displacements and slopes, the *SS* and *AA* modes correspond to $\bar{n}$ being even and the *AS* and *SA* mode correspond to $\bar{n}$ being odd.

The above five coupled ordinary differential equations and the associated boundary conditions can be discretized and these constitute the eigenvalue problem of the form,

$$[A]\bar{x} = \omega[B]\bar{x} \quad (23)$$

where $[A]$ is a banded matrix with bandwidth 23, $[B]$ is a diagonal matrix, and $\bar{x}$ is the solution vector. For a given axial wave number m, the smallest eigenvalue is obtained using the inverse power method (Hui and Hansen, 1982) in conjunction with the use of the linear equation solver LINPACK (Dongarra et al., 1979). As expected, the fundamental frequencies of imperfect oval cylinders generally correspond to one axial half wave $(m=1)$ in the vibration mode.

3 Discussion of Results

Due to numerous material parameters involved in a laminated shell structure, all the example problems are

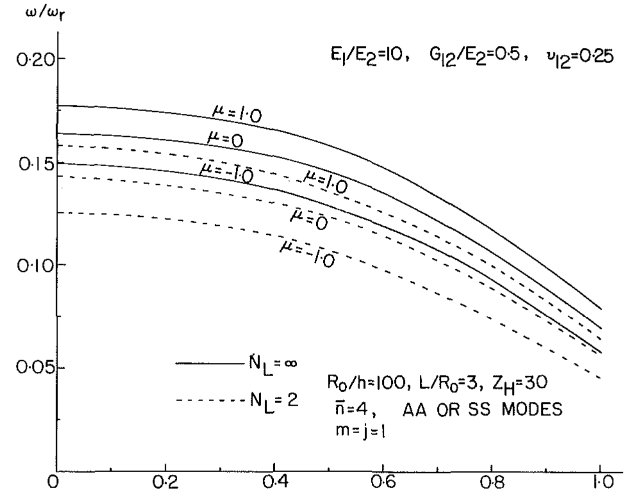

Fig. 1(*a*) Fundamental frequency versus the eccentricity parameter for anti-symmetric cross-ply Boron-epoxy, simply-supported oval cylindrical shells (*SS* and *AA* modes with $\bar{n}$ = even)

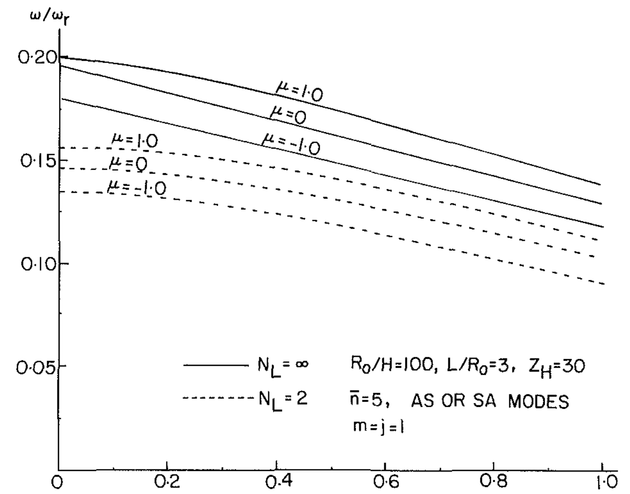

Fig. 1(*b*) Fundamental frequency versus the eccentricity parameter for anti-symmetric cross-ply Boron-epoxy, simply-supported oval cylindrical shells (*SA* and *AS* modes with $\bar{n}$ = odd)

restricted to antisymmetric (even number of equal-thickness layers and each layer is orthotropic) cross-ply oval cylindrical shells made of Boron-epoxy material where (Soldatos, 1984),

$$E_1/E_1 = 10, \quad G_{12}/E_2 = 0.5, \quad \nu_{12} = 0.25, \quad \nu_{21} = \nu_{12}E_2/E_1 \quad (24)$$

This implies zero stretching-twisting coupling so that $A_{16}=0$, $A_{26}=0$ ($A_{16}^*=0$, $A_{26}^*=0$), zero bending-twisting coupling so that $D_{16}=0$, $D_{26}=0$ ($D_{16}^*=0$, $D_{26}^*=0$). Further $B_{16}=0$, $B_{26}=0$ ($B_{16}^*=0$, $B_{26}^*=0$, $B_{61}^*=0$, $B_{62}^*=0$) and the nonzero bending-stretching coupling coefficients are B_{11}, B_{12}, B_{21}, B_{22}, and B_{66}.

Figure 1(*a*) shows a graph of the nondimensional fundamental frequency ω/ω_r versus the oval eccentricity parameter ϵ for antisymmetic, Boron-epoxy cross-ply oval cylindrical shells simply-supported at both ends. The number of layers N_L is infinite ($b_{ij}^*=0$) or two ($b_{ij}^*\neq 0$) as shown in solid or dotted lines, respectively. The geometric parameters are

$$R_o/h = 100, \quad L/R_o = 3 \quad Z_H = 30 \quad (25)$$

for a fixed value of $\bar{n}$ being an even integer, there exists a negligible difference in the fundamental frequencies between the *AA* and *SS* modes because their mode shapes are almost identical in shape except for a phase shift in the circumferential direction. The maximum fundamental frequency cor-

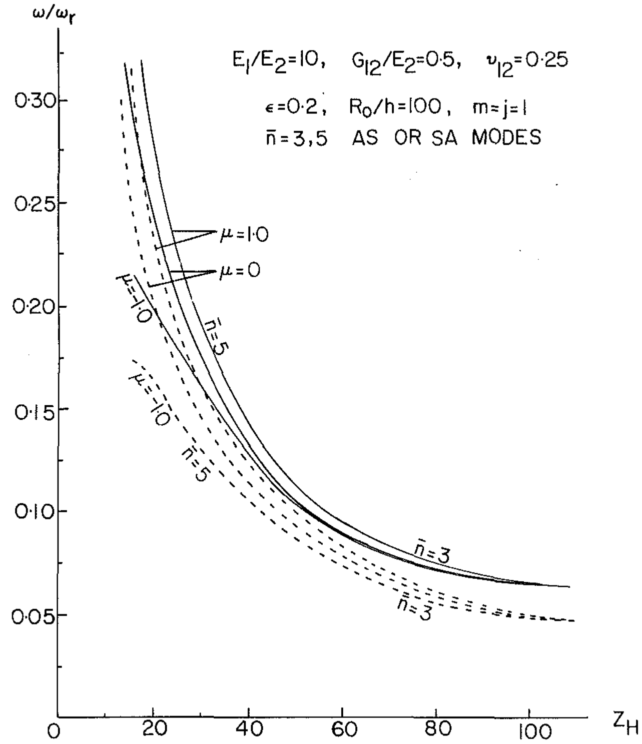

Fig. 2(*a*) Fundamental frequency versus the reduced-Badorf parameter for cross-ply Boron-epoxy, simply-supported oval cylindrical shells (*SS* and *AA* modes with $\bar{n}$ = even)

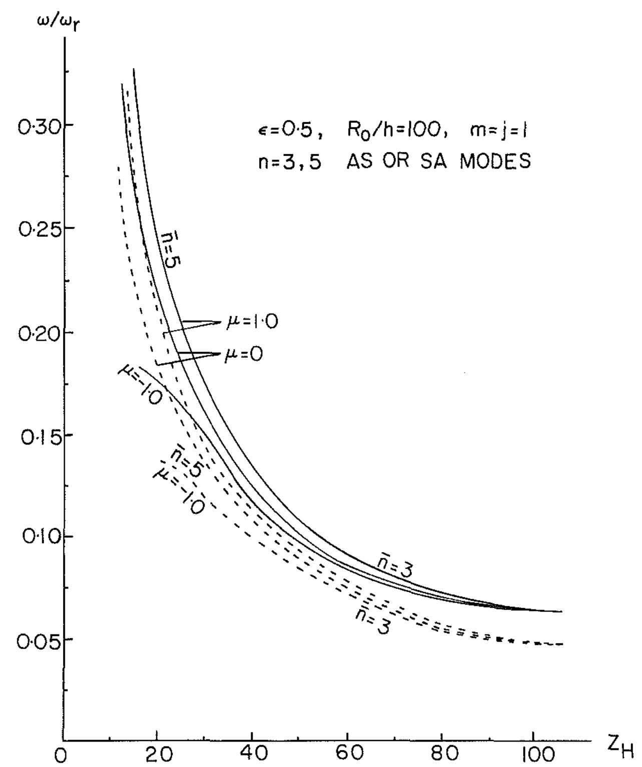

Fig. 2(*b*) Fundamental frequency versus the reduced-Batdorf parameter for cross-ply Boron-epoxy, simply-supported oval cylindrical shells (*SA* and *AS* modes with $\bar{n}$ = odd)

responds to a circular cylindrical shell ($\epsilon = 0$) and the frequencies decrease monotonically as the oval eccentricity ϵ increases. It can be seen that the presence of small axial geometric imperfection with one axial half wave ($j = 1$) may significantly raise or lower the fundamental frequencies for $\mu = 1$ and $= -1$, respectively. For positive imperfection amplitude, the shell bulges outwards prior to vibration, causing an additional increase of the curvature, and thus an increase of the fundamental frequencies. On the other hand, for negative values of the imperfection amplitudes, the shell bulges inwards causing a reduction of the curvature of the shell and thus a reduction of the frequencies. Significant reductions in the fundamental frequencies are found due to bending-stretching coupling by comparing the N_L = infinite and $N_L = 2$ curves.

The above effects of the geometric imperfections were qualitatively similar to the vibration behavior of simply supported cylindrical panels (Hui, 1984). Note that for even axial wave number ($j = m$ = even), the frequency of this nonfundamental mode does not depend on the sign of the imperfection amplitude. As a check on the analysis, the fundamental frequencies of the special case of a "perfect-laminated" and "perfect-isotropic homogeneous" simply-supported oval cylindrical shells agree with that presented by Soldatos (1984) and Kempner and Chen (1978), respectively.

The fundmental frequencies of the *SA* and *AS* modes are also indistinguishable and they correspond to $\bar{n} = 5$ as shown in Fig. 1(*b*), using the same material and geometric parameters as in Fig. 1(*a*).

Figure 2(*a*) shows a plot of fundamental frequency ($m = 1$) versus the reduced Batdorf parameter Z_H for antisymmetric cross-ply, simply-supported oval cylindrical shells for the *AS* and *SA* modes ($\bar{n}$ = 3 or 5). The following geometric parameters are kept fixed:

$$R_o/h = 100, \quad \epsilon = 0, \quad m = j = 1 \tag{26}$$

It can be seen that the presence of axial imperfections ($j = 1$) has very little effect on the fundamental frequencies for a large reduced Batdorf parameter ($Z_H = 70$). On the other hand, they may significantly increase or decrease the fundamental frequencies for a small reduced Batdorf parameter. The value of $\bar{n}$ becomes smaller for a larger reduced Batdorf parameter. Note that a large reduced Batdorf parameter corresponds to a longer or thinner shell. Comparing the N_L = infinite and $N_L = 2$ curves, significant reductions of the fundamental frequencies are seen to occur (due to bending-stretching coupling of the laminated shell) especially for a smaller reduced Batdorf parameter. Again, a positive imperfection amplitude raises the fundamental frequency while a negative value lowers it. The effects of the geometric imperfections diminish as the reduced Batdorf parameter increases.

A similar plot of the fundamental frequency versus the reduced Batdorf parameter for a larger oval eccentricity ϵ being 0.5 is presented in Fig. 2(*b*). The above discussions in Fig. 2(*a*) are also applicable in Fig. 2(*b*). However, for negative imperfection amplitude μ being -1.0, the fundamental frequency increases very slowly as the reduced Batdorf parameter is decreased. It actually reaches a peak at Z_H being approximately 20 and the frequency actually drops for smaller Z_H (not shown as this may be due to numerical instability).

Figure 3 shows a graph of the fundamental frequency versus the axial imperfection amplitude for antisymmetric cross-ply oval cylindrical shells. Only the *SA* and *AS* modes ($\bar{n}$ = 3, 7) are considered because the *SA* and *AA* modes ($\bar{n}$ = even) behave qualitatively in a similar way. The following parameters are held fixed:

$$R_o/h = 100, \quad \epsilon = 0.2 \tag{27}$$

It is found that the fundamental frequencies is significantly affected by the presence of an axial ($j = 1$) geometric imperfection for the reduced Batdorf parameter ($Z_H = 10$). However, the fundamental frequencies are relatively insensitive to axial geometric imperfection for a larger reduced Batdorf parameter ($Z_H = 40$ and $Z_H = 70$) for $N_L = 2$. For comparison

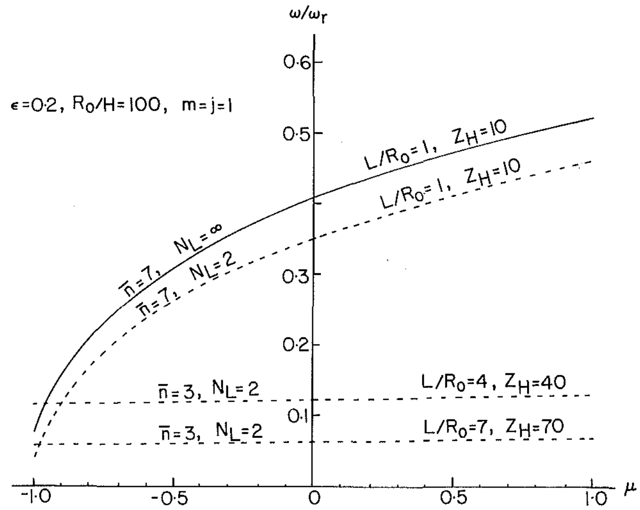

Fig. 3 Fundamental frequency versus the axial imperfection amplitude for cross-ply Boron-epoxy, simply-supported oval cylindrical shells (*SA* and *AS* modes with $\bar{n}$ = odd)

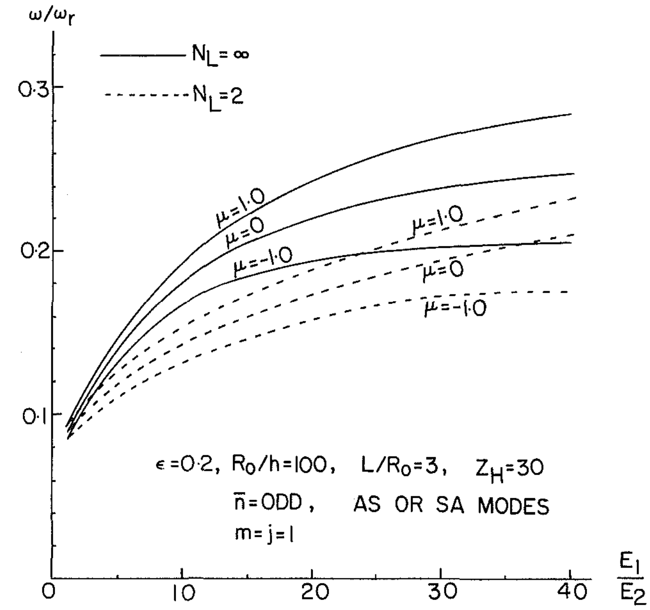

Fig. 4(*a*) Fundamental frequency versus Young's Moduli ratio for cross-ply Boron-epoxy, simply-supported oval cylindrical shells (*SS* and *AA* modes with $\bar{n}$ = even)

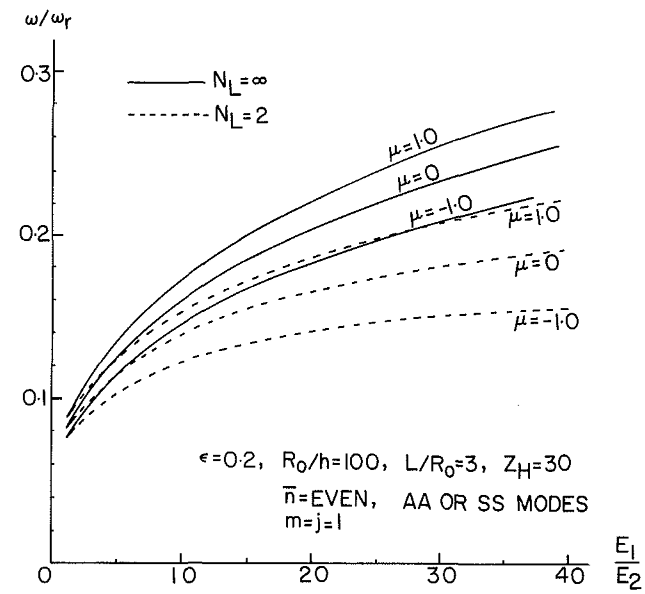

Fig. 4(*b*) Fundamental frequency versus Young's Moduli ratio for cross-ply Boron-epoxy, simply-supported oval cylindrical shells (*SA* and *AS* modes with $\bar{n}$ = odd)

purposes, the curves for the orthotropic shell (N_L + infinite) with $L/R_o = 1$, $Z_H = 10$, is also plotted in (solid lines).

Figure 4(*a*) shows a plot of the fundamental frequency versus Young's moduli ratio for antisymmetric cross-ply, simply-supported, oval cylindrical shells for the *AA* and *SS* modes ($\bar{n}$ = even). The fundamental frequencies for the *SS* and *AA* modes are almost identical. The frequencies increase with Young's moduli ratio and the increase in frequencies is slightly more pronounced for the orthotropic (N_L = infinite) than the $N_L = 2$ shell. Clearly, for $E_1/E_2 = 1$ (that is, for isotropic-homogeneous shell) the N_L = infinite and $N_L = 2$ curves coincide. A similar graph for the *AS* and *SA* modes ($\bar{n}$ = odd) is presented in Fig. 4(*b*) and the above comments are also qualitatively applicable to these modes. As predicted by Soldatos (1984), there exist negligible differences between the $\bar{n}$ = odd and $\bar{n}$ = even frequency curves.

4 Concluding Remarks

The influence of axial geometric imperfections on the fundamental vibration frequencies of cross-ply simply-supported oval cylindrical shells has been studied. The presence of such imperfection amplitudes of the order of the shell thickness may significantly raise or lower the fundamental frequencies, depending on the imperfection and the vibration mode wave numbers. Parameter variations involving the oval eccentricity, the number layers, the reduced-Batdorf parameter, and Young's moduli ratio were examined.

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