Imperfection-Sensitivity of Long Antisymmetric Cross-Ply Cylindrical Panels Under Shear Loads

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This paper examines the theoretical postbuckling behavior and imperfection-sensitivity of antisymmetrically laminated open, long, cylindrical panels under shear loads. The longitudinal edges may be simply-supported or clamped. It is found that the cylindrical panels may be sensitive to the presence of geometric imperfections depending primarily on the reduced-flatness parameter and Young's modulus ratio. The shear buckling load and the postbuckling coefficients are plotted as a function of the shell geometry, number of layers, and material parameters. The paper is the first in the literature to examine the postbuckling behavior of open laminated cylindrical panels under shear loads.

1 Introduction

Due to increasing use of light-weight, high-strength composite materials in the aerospace and mechanical engineering applications, shear buckling of laminated plates has received increased attention over the past twenty years. Ashton and Love (1969), Whitney (1969), Chamis (1971), Housner and Stein (1975), and Hui (1984b), among others, have studied shear buckling of composite rectangular plates. Excellent reviews on this subject were written by Johns (1971) and Leissa (1985). The "postbuckling" behavior of laminated rectangular plates under shear loads has also been examined by a number of authors. For example, Agarwal (1981), Sumihara et al. (1981), Zhang and Matthews (1981, 1984), and Stein (1985a,b) have examined these postbuckling problems in recent years and some experiments on diagonal tension behavior of shear panels were performed by Kaminski and Ashton (1971). Stein (1985a,b) found that laminated long plates may carry loads which are considerably higher than the classical shear buckling loads. The effects of the direction of shear load on the postbuckling behavior of these anisotropic rectangular plates were studied by Zhang and Matthews (1981, 1984). General computer codes were written by Viswanathan et al. (1974) and Bauld and Satyamurthy (1979).

Shear buckling of generally laminated open "cylindrical" panels have been examined by Zhang and Matthews (1983a) using beam functions. The effects of shear loads on antisymmetrically laminated cross-ply cylindrical panels simply-supported along all four edges were presented by Hui (1987). Zhang and Matthews (1983b) also investigated the postbuckling behavior of laminated cylindrical panels under axial compression but not shear load. Surprisingly, it appears that thorough theoretical studies on the postbuckling behavior of laminated open cylindrical panels under shear loads have not been reported in the open literature, even in the important special case of isotropic-homogeneous cylindrical panels. Based on some experiments (Rafel, 1943) and preliminary calculations on isotropic-homogeneous cylindrical panels under shear loads, it may be conjectured that cylindrical panels are still sensitive to the presence of geometric imperfections as in the case of axial load (Koiter, 1956; Stephens, 1971; Hui et al., 1981; Hui, 1985) but possibly to a lesser extent. Thus it is of interest to study the postbuckling and imperfection-sensitivity of laminated cylindrical panels under shear loads.

The objective of this paper is to examine the postbuckling behavior and imperfection-sensitivity of laminated open cylindrical panels under shear loads. Koiter's theory of elastic stability is employed and the analysis is based on a solution of the nonlinear Donnell-type equilibrium and compatibility equations. Using Koiter's perturbation technique, a sequence of linear ordinary differential equations are derived for the buckling and postbuckling state, using the simplifying assumption that the cylindrical panels are infinitely long, thus permitting a separable form of the solution. The buckling differential equations are discretized using a central difference scheme and the resulting eigenvalue problem is solved using the shifted inverse power method. Likewise, the second order ordinary differential equations are discretized and solved us-

Contributed by the Applied Mechanics Division for presentation at the 1987 Applied Mechanics, Biomechanics, and Fluids Engineering Conference, Cincinnati, OH, June 14–17, 1987, of the American Society of Mechanical Engineers.

Discussion of this paper should be addressed to the Editorial Department, ASME, United Engineering Center, 345 East 47th Street, New York, N.Y. 10017, and will be accepted until two months after final publication of the paper itself in the JOURNAL OF APPLIED MECHANICS. Manuscript received by ASME Applied Mechanics Division, April 1, 1986; final revision, October 6, 1986.

Paper No. 87-APM-28.

ing Gaussian elimination. The shear buckling load and the associated postbuckling coefficients are presented as a function of the geometric and material parameters of the laminated cylindrical panels. Example problems are chosen from antisymmetrically laminated cross-ply long cylindrical panels, simply-supported or clamped along the longitudinal edges. Parameter variation involving the geometry of the shell specified by the simplified-flatness parameter and the material parameter involving Young's modulus ratio will be presented. Particular attention will be given to Boron-epoxy and isotropic-homogeneous cylindrical panels.

It is found that open laminated cylindrical panels under shear loads have stable postbuckling behavior (not sensitive to geometric imperfections) for sufficiently small values of the simplified-flatness parameter. For larger values of the reduced-flatness parameter, the shear buckling loads of the cylindrical panels are found to be sensitive to the presence of geometric imperfections. This is especially true for Young's modulus ratio close to unity. Long cylindrical panels with clamped edges are less imperfection-sensitive than those with simply-supported edges.

2 Governing Differential Equations and Classical Buckling Load

The governing Donnell-type nonlinear equilibrium and compatibility equations for generally laminated cylindrical panels are, in nondimensional form, respectively, Hui (1985, 1986b)

$$L_{d^*}(w) + L_{b^*}(f) + f_{,xx} = f_{,yy}w_{,xx} + f_{,xx}w_{,yy} - 2f_{,xy}w_{,xy} \quad (1)$$

$$L_{a^*}(f) - L_{b^*}(w) - w_{,xx} = (w_{,xy})^2 - w_{,xx}w_{,yy} \quad (2)$$

where,

$$w = W/h,\ f = F/(E_2h^3),\ (x,y) = (X,Y)/(Rh)^{1/2}$$
$$a^*_{ij} = (E_2h)A^*_{ij},\ b^*_{ij} = B^*_{ij}/h,\ d^*_{ij} = D^*_{ij}/(E_2h^3) \quad (3)$$

In the above, W is the out-of-plane displacement (positive outwards), F is the stress function, R is the shell radius, h is the total thickness of the laminated shell, E_2 is the Young's modulus in the in-plane transverse direction, and the $L_{a^*}(\ )$, $L_{b^*}(\ )$, and $L_{d^*}(\ )$ are the nondimensional linear differential operators defined by Hui (1985, 1986) and Tennyson et al. (1971).

Using Koiter's (1945) theory of elastic stability, the total displacement and the total stress function can be expanded in the perturbed form,

$$w = w_p + \xi w_I + \xi^2 w_{II}$$
$$f = f_p + \xi f_I + \xi^2 f_{II} \quad (4)$$

where ξ is the amplitude of the normalized buckling mode. The axial compressive load σ, the lateral pressure p and the torsional load τ are related to the prebuckling stress function f_p by,

$$(\sigma,p,\tau) = (f_{p,yy}, f_{p,xx}, -f_{p,xy}) = (N_x,N_y,N_{xy})R/(E_2h^2) \quad (5)$$

where N_x, N_y, and N_{xy} are the membrane stress resultants.

Substituting the total displacement and stress function into the governing nonlinear differential equations and collecting terms which involve ξ, one obtains,

$$L_{d^*}(w_I) + L_{b^*}(f_I) + f_{I,xx} + \sigma_x w_{I,xx} + \sigma_y w_{I,yy} - 2\tau w_{I,xy} - w_{p,xx}f_{I,yy} - w_{p,yy}f_{I,xx} + 2w_{p,xy}f_{I,xy} = 0 \quad (6)$$

$$L_{a^*}(f_I) - L_{b^*}(w_I) - w_{I,xx} - 2w_{p,xy}w_{I,xy} + w_{p,xx}w_{I,yy} + w_{p,yy}w_{I,xx} = 0 \quad (7)$$

For an antisymmetric cross-ply laminate, the nondimensional linear operators are defined to be

$$L_{a^*}(\ ) = a^*_{22}(\ )_{,xxxx} + (2a^*_{12} + a^*_{66})(\ )_{,xxyy} + a^*_{11}(\ )_{,yyyy}$$
$$L_{b^*}(\ ) = b^*_{21}(\ )_{,xxxx} + (b^*_{11} + b^*_{22} - 2b^*_{66})(\ )_{,xxyy} + b^*_{12}(\ )_{,yyyy}$$
$$L_{d^*}(\ ) = d^*_{11}(\ )_{,xxxx} + (2)(d^*_{12} + 2d^*_{66})(\ )_{,xxyy} + d^*_{22}(\ )_{,yyyy}$$

Assuming that the cylindrical panel is sufficiently long so that the boundary conditions at the two curved edges can be neglected, the buckling mode can be written in the separable form,

$$w_I(x,y) = w_c(y)\cos(Mx) + w_s(y)\sin(Mx)$$
$$f_I(x,y) = f_c(y)\cos(Mx) + f_s(y)\sin(Mx) \quad (8)$$

The wave number M is defined as,

$$M = m\pi(Rh)^{1/2}/L \quad \text{or} \quad M = (m\pi/\theta_s)(B/L) \quad (9)$$

where B is the curved distance between the two longitudinal edges and the simplified-flatness parameter θ_s is,

$$\theta_s = B/(Rh)^{1/2} \quad (10)$$

Substituting the separable form of the buckling mode $w_I(x,y)$ and $f_I(x,y)$ into the equilibrium equation for the buckling state, and collecting terms which involve cos (Mx) and sin (Mx), one obtains, respectively,

$$\begin{aligned}
&d^*_{22}w_c(y)_{,yyyy} + [\sigma_y - 2M^2(d^*_{12} + 2d^*_{66})]w_c(y)_{,yy} \\
&+ (-\sigma_x M^2 + d^*_{11}M^4)w_c(y) + (4d^*_{26}M)w_s(y)_{,yyy} \\
&- (4d^*_{16}M^3 + 2\tau M)w_s(y)_{,y} + b^*_{12}f_c(y)_{,yyyy} \\
&- [(b^*_{11} + b^*_{22} - 2b^*_{66})(M^2) + w_{p,xx}]f_c(y)_{,yy} \\
&+ (-M^2 + b^*_{21}M^4 + M^2 w_{p,yy})f_c(y) \\
&+ (2b^*_{16} - b^*_{62})Mf_s(y)_{,yyy} + [(2b^*_{26} - b^*_{61})(-M^3) \\
&+ 2Mw_{p,xy}]f_s(y)_{,y} = 0
\end{aligned} \quad (11)$$

$$\begin{aligned}
&d^*_{22}w_s(y)_{,yyyy} + [\sigma_y - (2M^2)(d^*_{12} + 2d^*_{66})]w_s(y)_{,yy} \\
&+ (d^*_{11}M^4 - M^2\sigma_x)w_s(y) - (4M)d^*_{26}w_c(y)_{,yyy} \\
&+ (4d^*_{16}M^3 + 2\tau M)w_c(y)_{,y} + b^*_{12}f_s(y)_{,yyyy} \\
&+ [(b^*_{11} + b^*_{22} - 2b^*_{66})(-M^2) - w_{p,xx}]f_s(y)_{,yy} \\
&+ (b^*_{21}M^4 - M^2 + w_{p,yy}M^2)f_s(y) \\
&+ (2b^*_{16} - b^*_{62})(-M)f_c(y)_{,yyy} + [(2b^*_{26} - b^*_{61})(M^3) \\
&- (2M)w_{p,xy}]f_c(y)_{,y} = 0
\end{aligned} \quad (12)$$

Similarly, collecting terms involving cos (Mx) and sin (Mx) in the compatibility equation for the buckling state, one obtains, respectively,

$$\begin{aligned}
&a^*_{11}f_c(y)_{,yyyy} + (2a^*_{12} + a^*_{66})(-M^2)f_c(y)_{,yy} \\
&+ a^*_{22}M^4 f_c(y) - 2a^*_{16}Mf_s(y)_{,yyy} + 2a^*_{26}M^3 f_s(y)_{,y} \\
&- b^*_{12}w_c(y)_{,yyyy} + [(b^*_{11} + b^*_{22} - 2b^*_{66})M^2 \\
&+ w_{p,xx}]w_c(y)_{,yy} + [-b^*_{21}M^4 + M^2 - w_{p,yy}M^2]w_c(y) \\
&- (2b^*_{16} - b^*_{62})(M)w_s(y)_{,yyy} \\
&+ [(2b^*_{26} - b^*_{61})M^3 - 2Mw_{p,xy}]w_s(y)_{,y} = 0
\end{aligned} \quad (13)$$

$$\begin{aligned}
&a^*_{11}f_s(y)_{,yyyy} + (2a^*_{12} + a^*_{66})(-M^2)f_s(y)_{,yy} \\
&+ a^*_{22}M^4 f_s(y) - (2a^*_{16})(-M)f_c(y)_{,yyy} \\
&- (2a^*_{26})(M^3)f_c(y)_{,y} - b^*_{12}w_s(y)_{,yyyy} \\
&+ [(b^*_{11} + b^*_{22} - 2b^*_{66})M^2 + w_{p,xx}]w_s(y)_{,yy} \\
&+ (-b^*_{21}M^4 + M^2 - M^2 w_{p,yy})w_s(y) \\
&+ (2b^*_{16} - b^*_{62})(M)w_c(y)_{,yyy} \\
&- (2b^*_{26} - b^*_{61})M^3 w_c(y)_{,y} = 0
\end{aligned} \quad (14)$$

The boundary conditions at the two longitudinal edges of a laminated cylindrical panel are taken to be identical. The possible in-plane boundary conditions are

$$U(y=0)=0 \quad \text{or} \quad N_{xy}(y=0)=0$$
$$V(y=0)=0 \quad \text{or} \quad N_y(y=0)=0 \qquad (15)$$

However, in all the example problems chosen, the following three boundary conditions are enforced (weakest in-plane conditions),

$$f_{,xy}(y=0)=0$$
$$f_{,xx}(y=0)=0$$
$$w(y=0)=0 \qquad (16)$$

For clamped edges, one obtains, $w_{,y}(y=0) = 0$ whereas for simply-supported edges, the condition is $M_y(y=0) = 0$. The zero-amount condition implies

$$-b_{12}^* f_{,yy}(y=0) - b_{22}^* f_{,xx}(y=0)$$
$$+ b_{62}^* f_{,xy}(y=0) - d_{22}^* w_{,yy}(y=0)$$
$$-2d_{26}^* w_{,xy}(y=0)=0 \qquad (17)$$

For an antisymmetric cross-ply laminated panel ($b_{62}^* = 0$, $d_{26}^* = 0$) under pure shear, the membrane prebuckling state satisfies the above zero-moment condition. Likewise, for an antisymmetric angle-ply panel ($b_{12}^* = 0$, $b_{22}^* = 0$, $d_{26}^* = 0$) under axial compression or biaxial compression, the membrane prebuckling state also satisfies the $M_y(y=0) = 0$ requirement. The boundary conditions for the buckling state can be easily obtained by substituting the separable form of the buckling mode $w_I(x, y)$ and $f_I(x, y)$ into equations (16) and (17). The above four coupled linear ordinary differential equations with four unknown variables $w_s(y)$, $w_c(y)$, $f_s(y)$, and $f_c(y)$ and the associated boundary conditions are discretized using a central finite difference scheme using approximately 200 integration points between the two longitudinal edges (only 100 integration points are used for small θ_s). The resulting system of homogeneous linear algebraic equations can be expressed in matrix form,

$$[A]\vec{x} = \tau[B]\vec{x} \qquad (18)$$

which can be solved using a "shifted" inverse power method (Hui and Hansen (1982)). Similar to the case of shear buckling of antisymmetric cross-ply simply-supported rectangular plates (Hui 1984b), the buckling load is independent of the direction of the applied shear loads. Thus, the smallest eigenvalue occurs in positive and negative pairs so that the standard power method fails and it is necessary to use the "shifted" technique. The classical buckling load is obtained by searching for the smallest eigenvalue for all possible values of the "continuous" wave number M. The corresponding buckling mode is normalized such that the largest magnitude of the displacement is one, that is,

$$[w_s(y)^2 + w_c(y)^2]^{1/2} = 1 \qquad (19)$$

3 Second Order Fields

Substituting the total displacement and stress function (equations (4)) into the governing nonlinear partial differential equations and collecting terms which involve ξ^2, the equilibrium and compatibility equations for the second order fields are, respectively,

$$L_{d^*}(w_{II}) + L_{b^*}(f_{II}) + f_{II,xx} + \sigma_x w_{II,xx}$$
$$+ \sigma_y w_{II,yy} - 2\tau w_{II,xy} - w_{p,xx} f_{II,yy}$$
$$- w_{p,yy} f_{II,xx} + 2w_{p,xy} f_{II,xy}$$
$$= f_{I,yy} w_{I,xx} + f_{I,xx} w_{I,yy} - 2f_{I,xy} w_{I,xy} \qquad (20)$$

$$L_{a^*}(f_{II}) - L_{b^*}(w_{II}) - 2w_{p,xy} w_{II,xy}$$
$$+ w_{p,xx} w_{II,yy} + w_{p,yy} w_{II,xx} - w_{II,xx}$$
$$= (w_{I,xy})^2 - w_{I,xx} w_{I,yy} \qquad (21)$$

Upon substitution of the separable form of the buckling mode $w_I(x,y)$ and $f_I(x,y)$ into the right-hand sides of the equilibrium and compatibility equations for the second order fields, it is clear that $w_{II}(x,y)$ and $f_{II}(x,y)$ can be written in the separable form

$$w_{II}(x,y) = w^*(y) + w_A(y)\cos(2Mx) + w_B(y)\sin(2Mx)$$
$$f_{II}(x,y) = f^*(y) + f_A(y)\cos(2Mx) + f_B(y)\sin(2Mx) \qquad (22)$$

Substituting the separable forms of $w_I(x,y)$, and $f_I(x,y)$ and $w_{II}(x,y)$, $f_{II}(x,y)$ into the partial differential equations for the second order fields, one obtains two coupled ordinary differential equations in $w^*(y)$ and $f^*(y)$ and four uncoupled ordinary differential equations in $w_A(y)$, $w_B(y)$, $f_A(y)$, and $f_B(y)$. The two coupled equilibrium and compatibility equations for $w^*(y)$ and $f^*(y)$ are, respectively,

$$d_{22}^* w^*(y)_{,yyyy} + b_{12}^* f^*(y)_{,yyyy}$$
$$= (-M^2/2)\{[w_s(y)f_s(y)]_{,yy} + [w_c(y)f_c(y)]_{,yy}\} \qquad (23)$$

$$a_{11}^* f^*(y)_{,yyyy} - b_{12}^* w^*(y)_{,yyyy}$$
$$= (M^2/2)\{[w_s(y)w_s(y)_{,y}]_{,y} + [w_c(y)w_c(y)_{,y}]_{,y}\} \qquad (24)$$

The four coupled ordinary differential equations are, from the equilibrium equations collecting terms involving sin $(2Mx)$, respectively,

$$L_{11}(w_A, w_B, f_A, f_B) = (-M^2/2)\{[w_s(y)f_c(y)_{,yy}$$
$$+ w_s(y)_{,yy} f_c(y) - 2w_s(y)_{,y} f_c(y)_{,y}]$$
$$+ [w_c(y)f_s(y)_{,yy} + w_c(y)_{,yy} f_s(y)$$
$$-2w_c(y)_{,y} f_s(y)_{,y}]\} \qquad (25)$$

$$L_{12}(w_A, w_B, f_A, f_B) = (-M^2/2)\{[w_s(y)f_s(y)]_{,yy}$$
$$+ [w_c(y)f_c(y)]_{,yy}\} + (M^2/2)\{[w_s(y)f_s(y)_{,yy}$$
$$+ w_s(y)_{,yy} f_s(y) - 2w_s(y)_{,y} f_s(y)_{,y}]$$
$$- [w_c(y)f_c(y)_{,yy} + w_c(y)_{,yy} f_c(y)$$
$$-2w_c(y)_{,y} f_c(y)_{,y}]\} \qquad (26)$$

and from the compatibility equation by collecting terms involving sin $(2Mx)$ and cos $(2Mx)$, respectively,

$$L_{13}(w_A, w_B, f_A, f_B) = (M^2/2)[w_s(y)w_c(y)_{,yy}$$
$$+ w_c(y)w_s(y)_{,yy} - 2w_s(y)_{,y} w_c(y)_{,y}] \qquad (27)$$

$$L_{14}(w_A, w_B, f_A, f_B) = (M^2/2)[w_s(y)_{,y}{}^2$$
$$- w_s(y)w_s(y)_{,yy} - w_c(y)_{,y}{}^2$$
$$+ w_c(y)w_c(y)_{,yy}] \qquad (28)$$

In the above, $L_{11}(w_A, w_B, f_A, f_B)$, $L_{12}(w_A, w_B, f_A, f_B)$, $L_{13}(w_A, w_B, f_A, f_B)$, and $L_{14}(w_A, w_B, f_A, f_B)$ are obtained from the left-hand sides of the buckling ordinary differential equations (equations (11), (12), (13), and (14)) by replacing $w_c(y)$, $w_s(y)$, $f_c(y)$, $f_s(y)$ by $w_A(y)$, $w_B(y)$, $f_A(y)$, $f_B(y)$; further, the wave number M is replaced by $2M$ and τ is replaced by the classical buckling load τ_c. Since the in-plane boundary conditions for the second order fields do not involve $f^*(y)$ nor its derivative, an additional condition is needed to solve the (w^*, f^*) problem. The average second order longitudinal stress is set to zero (Hui, 1984a)

$$\int_{y=0}^{\theta_s} \int_{x=0}^{\infty} N_{x_{II}}(x,y)\,dx\,dy = 0 \qquad (29)$$

which implies

$$f^*_{,y}(y=0)=0 \quad \text{and} \quad f^*_{,y}(y=\theta_s)=0 \qquad (30)$$

Since the postbuckling coefficient (see next section) and the differential equation do not depend explicitly on $f^*(y)$ or $f^*(y)_{,y}$, it is only necessary to solve for $f^*(y)_{,yy}$. That is, a constant shift in $f^*(y)$ has no effect on the postbuckling coefficient so that one may arbitrarily set,

$$f^*(y=0)=0, \quad f^*(y=\theta_s)=0 \qquad (31)$$

The remaining boundary conditions for the clamped conditions at the longitudinal edges are,

$$w^*(y=0)=0, \qquad w^*_{,y}(y=0)=0,$$
$$w^*(y=\theta_s)=0, \qquad w^*_{,y}(y=\theta_s)=0, \tag{32}$$

For a simply-supported cylindrical panel, the above zero-slope conditions are replaced by the zero-moment condition,

$$-b^*_{12}f^*_{,yy}(y=0)-d^*_{22}w^*_{,yy}(y=0)=0 \tag{33}$$

with the same condition at $y=\theta_s$. Since the cylindrical panels are "open," the single-valuedness of the displacements need not, and in fact, should not be enforced.

The boundary conditions for the (w_A, w_B, f_A, f_B) problem are obtained from equations (16) and (17) by substituting the appropriate separable forms for the second order fields. The coupled ordinary differential equations and the associated boundary conditions are discretized using a central finite difference scheme and solved using a Gaussian elimination equation solver LINPACK (Dongarra et al., 1979).

4 Initial Postbuckling Behavior

The initial postbuckling behavior of a laminated cylindrical panel subjected to shear loads is examined using Koiter's (1945) theory of elastic stability. Koiter's theory was reformulated in terms of the mixed formulation involving the stress function by Budiansky and Hutchinson (1964). Within the assumptions employed by Koiter for a single-mode system, the structure will be able to carry the applied shear loads in the initial postbuckling stage (that is, not sensitive to geometric imperfections) if the postbuckling b coefficent is positive whereas the opposite is true if b is negative. The degree of imperfection-sensitivity is measured by the magnitude of the b coefficient. In passing, an alternative approach to measure imperfection-sensitivity is to compute the initial postbuckling slope (Stein 1968); however, this method will not be used in this paper.

The b coefficient is defined as (Hutchinson and Amazigo, 1967)

$$b=(R_1+R_2)/|d_1| \tag{34}$$

where,

$$R_1=(2)\int_{y=0}^{\theta_s}\int_{x=0}^{\infty}[(f_{I,xx}w_{I,y})w_{II,y} + (f_{I,yy}w_{I,x})w_{II,x}-f_{I,xy}(w_{I,x}w_{II,y} + w_{I,y}w_{II,x})]dx\,dy \tag{35}$$

$$R_2=\int_{y=0}^{\theta_s}\int_{x=0}^{\infty}[f_{II,xx}(w_{I,y})^2+f_{II,yy}(w_{I,x})^2 - 2f_{II,xy}(w_{I,x}w_{I,y})]dx\,dy \tag{36}$$

$$d_1=\int_{y=0}^{\theta_s}\int_{x=0}^{\infty}[\sigma_y(w_{I,y})^2+\sigma_x(w_{I,x})^2 - 2\tau w_{I,x}w_{I,y}]dx\,dy \tag{37}$$

Substituting the separable forms of $w_I(x,y)$, $f_I(x,y)$ and $w_A(x,y)$, $w_B(x,y)$, $f_A(x,y)$, $f_B(x,y)$ into the above expressions, one obtains,

$$\begin{aligned}R_1=(M^2)\int_{y=0}^{\theta_s}\{&[-w^*(y)_{,y}][f_s(y)w_s(y)_{,y}\\&+f_c(y)w_c(y)_{,y}]-(1/2)w_B(y)_{,y}[f_s(y)w_c(y)_{,y}\\&+f_c(y)w_s(y)_{,y}]-(1/2)w_A(y)_{,y}[-f_s(y)w_s(y)_{,y}\\&+f_c(y)w_c(y)_{,y}]+w_B(y)[f_s(y)_{,yy}w_c(y)\\&+f_c(y)_{,yy}w_s(y)]+w_A(y)[-f_s(y)_{,yy}w_s(y)\\&+f_c(y)_{,yy}w_c(y)]-w^*(y)_{,y}[f_s(y)_{,y}w_s(y)\\&+f_c(y)_{,y}w_c(y)]+(1/2)w_B(y)_{,y}[f_c(y)_{,y}w_s(y)\\&+f_s(y)_{,y}w_c(y)]+(1/2)w_A(y)_{,y}[-f_s(y)_{,y}w_s(y)\\&+f_c(y)_{,y}w_c(y)]-w_B(y)[f_c(y)_{,y}w_s(y)_{,y}\\&+f_s(y)_{,y}w_c(y)_{,y}]+w_A(y)[f_s(y)_{,y}w_s(y)_{,y}\\&-f_c(y)_{,y}w_c(y)_{,y}]\}dy\end{aligned} \tag{38}$$

$$\begin{aligned}R_2=(M^2)\int_{y=0}^{\theta_s}\{&(-2)f_B(y)[w_s(y)_{,y}w_c(y)_{,y}]\\&-f_A(y)[-w_s(y)_{,y}{}^2+w_c(y)_{,y}{}^2]\\&+(1/2)f^*(y)_{,yy}[w_s(y)^2+w_c(y)^2]\\&-(1/2)f_B(y)_{,yy}w_s(y)w_c(y)\\&+(1/4)f_A(y)_{,yy}[w_s(y)^2-w_c(y)^2]\\&-f_B(y)_{,y}[w_s(y)w_c(y)_{,y}+w_c(y)w_s(y)_{,y}]\\&+f_A(y)_{,y}[w_s(y)w_s(y)_{,y}-w_c(y)w_c(y)_{,y}]\}dy\end{aligned} \tag{39}$$

$$\begin{aligned}d_1=(1/2)\int_{y=0}^{\theta_s}\{&(\sigma_y)[w_s(y)_{,y}{}^2+w_c(y)_{,y}{}^2]\\&+(\sigma_x)(M^2)[w_s(y)^2+w_c(y)^2]-(2\tau)(M)[w_s(y)w_c(y)_{,y}\\&-w_c(y)w_s(y)_{,y}]\}dy\end{aligned} \tag{40}$$

The integrations are performed using Simpson's rule with approximately 200 integration points from $y=0$ to $y=\theta_s$.

5 Discussion of Results

The aim of this section is to present results for the initial postbuckling behavior of laminated cylindrical panels under shear loads. The cylindrical panels are assumed to be sufficiently long such that the boundary conditions at the two curved edges may be neglected. Example problems are chosen from Boron-epoxy laminated panels with the material parameters (Chia, 1980).

$$E_1/E_2=10, \quad G_{12}/E_2=1/3, \quad \nu_{12}=0.22 \tag{41}$$

The isotropic homogeneous special case can be obtained by setting,

$$E_1/E_2=1, \quad G_{12}/E_2=1/[2(1+\nu)], \quad \nu=0.3 \tag{42}$$

Within the assumptions of Donnell's shell theory, the present results are valid for all values of the radius to thickness ratio.

Figure 1(a) shows a graph of the nondimensional classical shear buckling load versus the simplified-flatness parameter for long isotropic homogeneous cylindrical panels simply-supported or clamped at the longitudinal edges. It should be noted that the variable τ^* is defined as,

$$\begin{aligned}\tau^*&=\tau_c\theta^2 \qquad \text{for } \theta<1\\ \tau^*&=\tau_c \qquad \text{for } \theta\geq 1\end{aligned} \tag{43}$$

The dimensional shear buckling load of a long flat plate is approximately the same as that for a cylindrical panel with a small curvature defined by $\theta_s < 1$. For large simplified-flatness parameter, the classical shear buckling load tends to level out (Hui, 1985). The optimum axial wave number ranges from ($M=2.39\pi$, $\theta_s=0.5$) to ($M=0.082\pi$, $\theta_s=8.0$) for clamped shells and from ($M=1.59\pi$, $\theta_s=0.5$) to ($M=0.078\pi$, $\theta_s=8.0$) for simply-supported shells.

Figure 1(b) shows a plot of the corresponding postbuckling b coefficient versus the simplified-flatness parameter for isotropic homogeneous, long cylindrical panels. As in the case of axial compression examined by Koiter (1956) and Stephens (1971), the b coefficient is positive for sufficiently small panel curvature. The transition values at which the b coefficient is zero occurs at approximately $\theta_s=2.65$ (or $\theta=0.766$) for simply-supported cylindrical panel and at $\theta_s=2.90$ (or $\theta=0.839$) for clamped panel where according to Koiter (1956)

$$\theta=\theta_s(2c)^{1/2}/(2\pi), \quad c=[3(1-\nu^2)]^{1/2} \tag{44}$$

The cylindrical panel appears to be most imperfection-

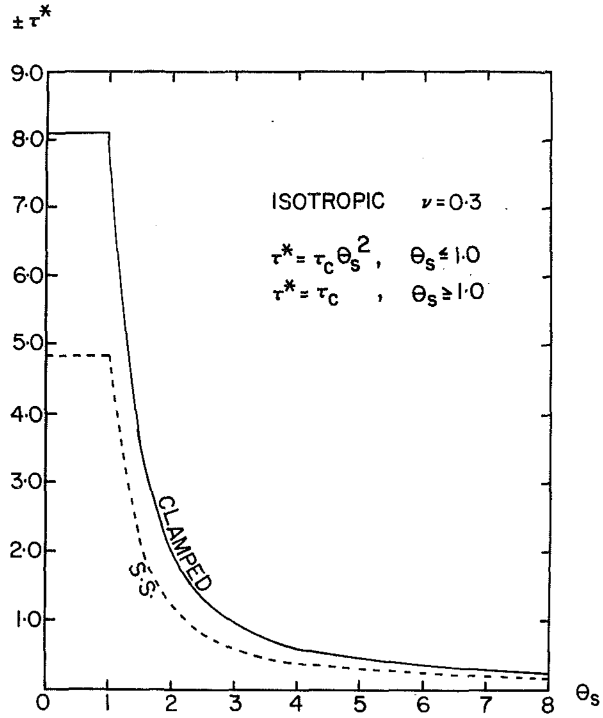

Fig. 1(*a*) Shear buckling load versus the reduced-flatness parameter for isotropic-homogeneous open cylindrical panels (ν = 0.3)

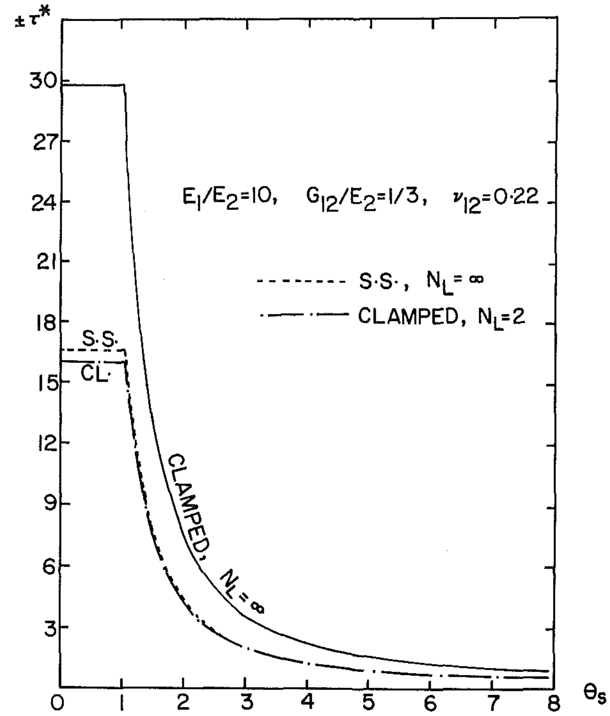

Fig. 2(*a*) Shear buckling load versus the reduced-flatness parameter for antisymmetric cross-ply boron-epoxy cylindrical panels (ν = 0.3)

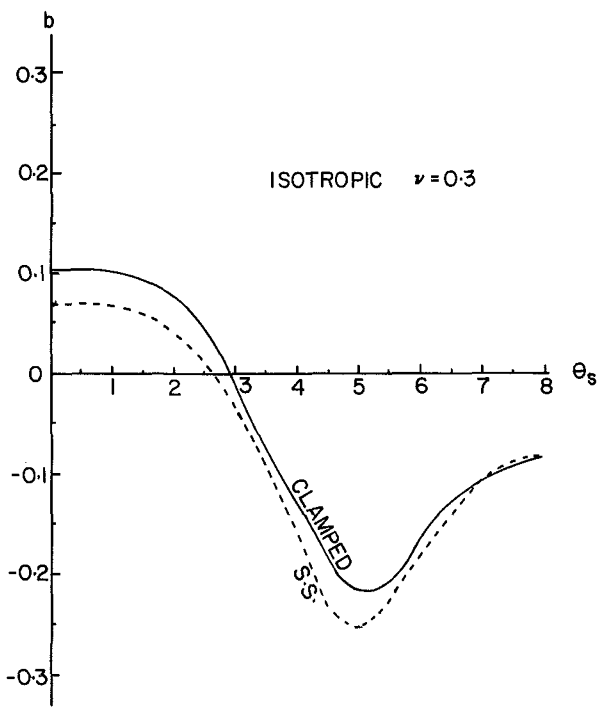

Fig. 1(*b*) Postbuckling *b* coefficient versus the reduced-flatness parameter for isotropic-homogeneous open cylindrical panels (ν = 0.3)

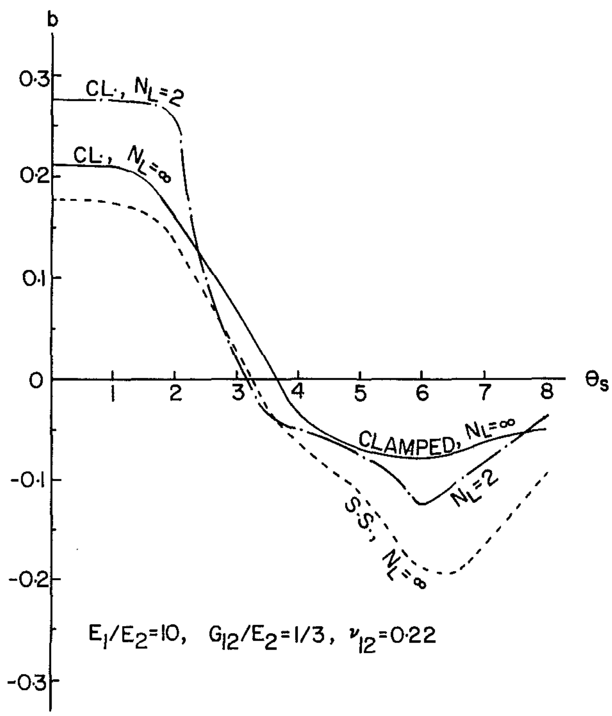

Fig. 2(*b*) Postbuckling *b* coefficient versus the reduced-flatness parameter for antisymmetric cross-ply boron-epoxy cylindrical panels

sensitive near $\theta_s = 5.0$ for both the simply-supported or the clamped cases. The cylindrical panels are slightly sensitive to geometric imperfections for large values of θ_s.

Figure 2(*a*) shows a graph of the classical shear buckling load versus the simplified-flatness parameter for antisymmetrically laminated cross-ply cylindrical panels. The behavior is similar to the isotropic homogeneous case except that the classical buckling load is much higher. Significant reductions in τ_c are found by comparing the two-layer design ($N_L = 2$) with the infinite-layer (N_L = infinite) orthotropic cylindrical panels. The shear buckling loads are in qualitative agreement with that presented by Hui (1986b). The optimum axial wave number M ranges from ($M = 2.80$, $\theta_s = 0.5$) to ($M = 0.19$, $\theta_s = 8.0$) for clamped shells with $N_L = \infty$ and ($M = 2.00$, $\theta_s = 0.5$) to ($M = 0.054$, $\theta_s = 8.0$) for simply-supported shells with $N_L = \infty$. For clamped shells with $N_L = 2$, M varies from ($M = 2.40$, $\theta_s = 0.05$) to ($M = 0.20$, $\theta_s = 8.0$).

The corresponding postbuckling b coefficient is depicted in Fig. 2(*b*). The b coefficient, for Boron-epoxy cylindrical panels are generally much more positive than that for isotropic homogeneous cylindrical panels, especially for small values of

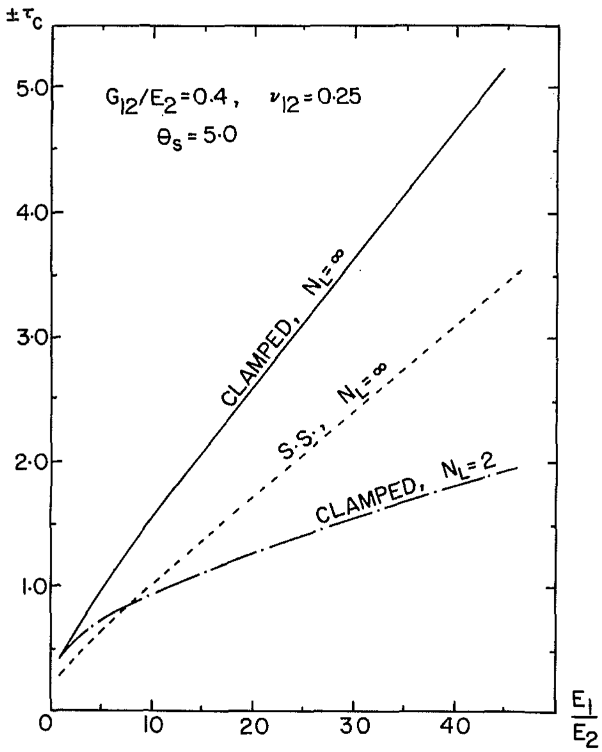

Fig. 3(*a*) Shear buckling load versus Young's modulus ratio for antisymmetric cross-ply cylindrical panels with $\theta_s = 5.0$

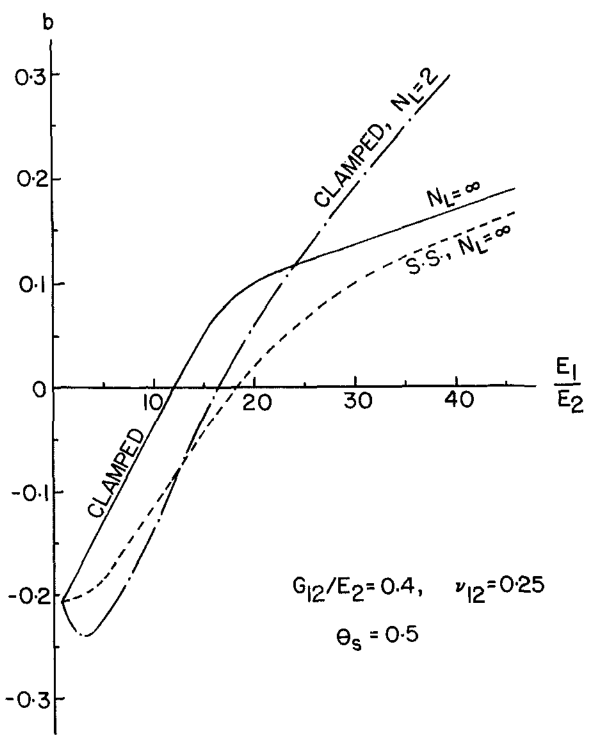

Fig. 3(*b*) Postbuckling *b* coefficient versus Young's modulus for antisymmetric cross-ply cylindrical panels with $\theta_s = 5.0$

θ_s. The transition values of the simplified-flatness parameters which correspond to $b = 0$ occur at $\theta_s = 3.7$ for (N_L = infinite, clamped), $\theta_s = 3.4$ for ($N_L = 2$ clamped) or (N_L = infinite, simply-supported). The cylindrical panel is most imperfection-sensitive near $\theta_s = 6.0$. Finally, it should be noted that the classical shear buckling loads and the postbuckling b coefficients are the same (within plotting accuracy) between the two-layered designs with (0 deg inside, 90 deg outside) or (90 deg inside, 0 deg outside). It should be noted that shear buckling loads of "symmetrically laminated" graphite-epoxy angle-ply cylindrical panels simply supported along all four edges depend a great deal on the direction of application of the shear load (Whitney, 1984). However, for the present "antisymmetrically laminated" cross-ply cylindrical panels, the sign of the shear loads has no effect on the buckling and postbuckling problems.

Finally, a plot of the classical shear buckling load versus Young's modulus ratio is depicted in Fig. 3(*a*). The fixed parameters are,

$$G_{12}/E_2 = 0.4, \quad \nu_{12} = 0.25, \quad \theta_s = 5.0 \tag{45}$$

The shear buckling load is roughly proportional to the Young's modulus ratio over most of the range of E_1/E_2 being considered. Again, the shear buckling loads are found to reduce significantly for a two-layer design due to bending-stretching coupling behavior of the laminated panel.

The corresponding postbuckling b coefficient versus E_1/E_2 is shown in Fig. 3(*b*). It appears that the increase in the shear buckling load due to higher values of E_1/E_2 is "not" accompanied by a higher degree of imperfection-sensitivity. In fact, the b coefficients are positive for E_1/E_2. Surprisingly, the $N_L = 2$ design corresponds to a relatively large value of the postbuckling b coefficient for sufficiently large values of E_1/E_2.

Concluding Remarks

The postbuckling and imperfection-sensitivity of long, antisymmetric cross-ply cylindrical panels (simply-supported or clamped at the longitudinal edges) under shear loads have been examined. The shear buckling loads of typical laminated cylindrical panels have been computed as a function of the reduced-flatness parameter and Young's modulus ratio. For sufficiently small values of the reduced-flatness parameter, the shear buckling loads of cylindrical panels are not sensitive to the presence of geometric imperfections. This agrees qualitatively with the postbuckling results of long laminated rectangular plates investigated by Stein (1985a, b). Over most of the practical range of the reduced-flatness parameter, especially for Young's modulus ratio relatively close to unity, the cylindrical panels are found to be postbuckling-unstable, hence imperfection-sensitive. This paper is the first to present a rigorous theoretical postbuckling analysis of laminated cylindrical panels under shear loads and this topic has not been dealt with in the open literature, even in the typical case of isotropic-homogeneous cylindrical panels.

Acknowledgment

The authors would like to acknowledge the generous use of the computer resources through ADML in the Ohio State University Robinson Laboratory Graphic Facilities.

References

Agarwal, B. L., 1981, "Postbuckling Behavior of Composite Shear Webs," *AIAA Journal*, Vol. 19, No. 7, pp. 933–939.

Ashton, J. E., and Love, T. S., 1969, "Shear Stability of Laminated Anisotropic Plates, Composite Materials: Testing and Design," American Society of Testing and Materials, ASTM-STP 460, pp. 352–361.

Bauld, N. R., Jr., and Satyamurthy, K., 1979, "Collapse Load Analysis for Plates and Panels," Air Force Flight Dynamics Lab., AFFDL-TR-79-3038, May.

Budiansky, B., and Hutchinson, J. W., 1964, "Dynamic Buckling of Imperfection-Sensitive Structures," *Proc. of the 11th International Congress of Applied Mechanics*, Munich, W. Germany, Gortler, H., ed., Springer-Verlag, 1966, pp. 636–651.

Chamis, C. C., 1971, "Theoretical Buckling Loads of Boron/Aluminum and Graphite/Resin Fiber Composite Anisotropic Plates," NASA TN D-6572, Dec.

Chia, C. Y., 1980, *A Nonlinear Analysis of Plates*, McGraw-Hill, New York.

Dongarra, J. J., Bunch, J. R., Moler, C. B., and Stewart, G. W., 1979, "LINPACK User's Guide," published by Society of Industrial and Applied

Math., Philadelphia; see also Argonne National Lab. Report TM-313, Aug. 1977.

Housner, J. M., and Stein, M., 1975, "Numerical Analysis and Parameter Studies of the Buckling of Composite Orthotropic Comp. and Shear Panels," NASA TN D-7996.

Hui, D., and Hansen, J. S., 1982, "Effect of Stringer Torsional Rigidity on Buckling of Integrally Stiffened Angle Ply Plates," *Fibre Science and Technology*, Vol. 16, No. 1, pp. 39-42.

Hui, D., Tennyson, R. C., and Hansen, J. S., 1981, "Mode Interaction of Axially Stiffened Cylindrical Shells: Effects of Stringer Axial Stiffness, Torsional Rigidity and Eccentricity," ASME Journal of Applied Mechanics, Vol. 48, No. 4, pp. 915-922.

Hui, D., 1984b, "Effects of Mode Interaction on Collapse of Short, Imperfect Thin-Walled Columns,' ASME Journal of Applied Mechanics, Vol. 50, No. 3, pp. 566-573.

Hui, D., 1984b, "Shear Buckling of Antisymmetric Cross-Ply Rectangular Plates," *Fibre Science and Technology*, Vol. 21, No. 4, pp. 327-340.

Hui, D., 1985, "Asymmetric Postbuckling of Symmetrically Laminated Cross Ply, Short Cylindrical Panels under Compression," *Composite Structures*, Vol. 3, pp. 81-95.

Hui, D., 1986, "Imperfection Sensitivity of Axially Compressed Laminated Flat Plates due to Bending-Stretching Coupling," *International Journal of Solids and Structures*, Vol. 22, No. 1, pp. 13-22.

Hui, D., 1987, "Effects of Shear Loads on Vibration and Buckling of Antisymmetic Cross-Ply Cylindrical Panels," *International Journal of Non-Linear Mechanics*, in press.

Hutchinson, J. W., and Amazigo, J. C., 1967, "Imperfection-Sensitivity of Eccentrically Stiffened Cylindrical Shells," *AIAA Journal*, Vol. 5, pp. 392-401.

Johns, D. J., 1971, "Shear Buckling of Isotropic and Orthotropic Plates – A Review," British Aeronautical Research Council, Rep. Memo R. and M. No. 3677.

Kaminski, B. E., and Ashton, J. E., 1971, "Diagonal Tension Behavior of Boron/Epoxy Shear Panels," *Journal of Composite Materials*, Vol. 5, Oct., pp. 553-558.

Koiter, W. T., 1945, "On the Stability of Elastic Equilibrium," Doctoral Thesis, Delft, The Netherlands, English Translation: NASA-TT-F10, 833, 1967, and AFFDL-TR-70-25, 1970.

Koiter, W. T., 1956, "Buckling and Post-Buckling Behaviour of Cylindrical Panel under Axial Compression," National Aeronautical Research Institute, Amsterdam, Report S-476, May, 14 pp.

Leissa, A. W., 1985, "Buckling of Laminated Composite Plates and Shell Panels," Air Force Wright Aeronautical Lab., AFWAL-TR-85-3069, 454 pp.

Rafel, N., 1943, "Effect of Normal Pressure on the Critical Shear Stress of Curved Sheet," NACA Wartime Report L-416, originally issued Jan. 1943 as restricted bulletin.

Stein, M., 1968, "Some Recent Advances in the Investigation of Shell Buckling," *AIAA Journal*, Vol. 6, No. 12, pp. 2339-2345.

Stein, M., 1985a, "Postbuckling of Long Orthotropic Plates in Combined Shear and Compression," *AIAA Journal*, Vol. 23, No. 5, pp. 788-794.

Stein, M., 1985b, "Analytical Results for Post-Buckling Behaviour of Plates in Compression and in Shear," Aspects of the Analysis of Plate Structures, a volume in honour of W. H. Wittrick, Dawe, D. J., Horsington, R. W., Kamtekar, A. G., and Little, G. H., eds., University of Birmingham, Clarendon Press, Oxford, pp. 205-223.

Stephens, W. B., 1971, "Imperfection-Sensitivity of Axially Compressed Stringer Reinfored Cylindrical Panels under Internal Pressure," *AIAA Journal*, Vol. 9, No. 9, pp. 1713-1719.

Sumihara, K., Kobayashi, S., and Koyama, K., 1981, "Shear Buckling Strength of CFRP Laminated Panels, 1st Report," *Journal of the Japan Society for Aeronautical and Space Sciences*, Vol. 29, June, pp. 321-332.

Tennyson, R. C., Chan, K. H., and Muggeridge, D. B., 1971, "The Effect of Axisymmetric Shape Imperfections on the Buckling of Laminated Anisotropic Circular Cylinders," *Canadian Aeronautics and Space Institute (CASI) Transactions*, Vol. 4, No. 2, Sept., pp. 131-139.

Viswanathan, A. V., Tamekuni, M., and Baker, L. L., 1974, "Elastic Stability of Laminated, Flat and Curved, Long Rectangular Plates Subjected to Combined Inplane Loads," NASA CR-2330, June.

Whitney, J. M., 1969, "Shear Buckling of Unsymmetrical Cross-Ply Plates," *Journal of Composite Materials*, Vol. 3, pp. 359-363.

Whitney, J. M., 1984, "Buckling of Anisotropic Laminated Cylindrical Plates," *AIAA Journal*, Vol. 22, No. 11, pp. 1641-1645.

Zhang, Y., and Matthews, F. L., 1981, "Postbuckling Behaviour of Cylindrically Curved Panels of Generally Layered Composite Materials with Small Initial Imperfections of Geometry," *Composite Structures*, Marshall, I. H., ed., Applied Science Publisher, London, England, pp. 428-441.

Zhang, Y., and Matthews, F. L., 1983a, "Initial Buckling of Curved Panels of Generally Layered Composite Materials," *Composite Structures*, Vol. 1, No. 1, pp. 3-30.

Zhang, Y., and Matthews, F. L., 1983b, "Postbuckling Behaviour of Curved Panels of Generally Layered Composite Materials," *Composite Structures*, Vol. 1, No. 2, pp. 115-135.

Zhang, Y., and Matthews, F. L., 1984, "Postbuckling Behavior of Anisotropic Laminated Plates under Pure Shear and Shear Combined with Compressive Loading," *AIAA Journal*, Vol. 22, No. 2, pp. 281-286.