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EFFECTS OF SHEAR LOADS ON VIBRATION AND BUCKLING OF ANTISYMMETRIC CROSS-PLY CYLINDRICAL PANELS

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Abstract—This work deals with the effects of shear loads on the vibration and buckling of typical antisymmetric cross-ply thin cylindrical panels, subjected to combined loads. Changes in the buckling loads due to geometric and material parameter variations are investigated with particular emphasis on distinguishing the symmetric and antisymmetric modes. The paper presents the first known results on shear buckling of cross-ply cylindrical panels as well as vibrations of these structures under shear loads. The resulting interaction curves will allow one to formulate effectively a preliminary design of these panels which will withstand shear loads.

1. INTRODUCTION

Buckling of thin laminated cylindrical panels has been a subject of considerable interest in recent years due to the increasing use of light-weight, high-strength materials in the aerospace industries. Cylindrical panels are one of the most frequently encountered load-carrying structural components in aircraft and other vehicles. It is evident from thorough reviews by Almroth [1], Almroth *et al.* [2] and Leissa [3, 4] and two texts on the mechanics of composite materials by Jones [5] and Chia [6] that buckling of laminated cylindrical panels under in-plane shear loads has received very little attention. Some very important preliminary results on shear buckling and postbuckling of generally layered composite cylindrical panels were recently reported by Zhang and Matthews [7, 8]; general computer codes were written by Viswanathan *et al.* [9] and Bauld and Satyamurthy [10]. In passing, theoretical studies on shear buckling of isotropic homogeneous cylindrical panels were reported by Batdorf *et al.* [11], Schildcrout and Stein [12] and Simitses and Sheinman [13] and some experimental data were presented by Rafel [14].

The shear buckling of composite cylindrical panels reported by Zhang and Matthews [7] should be viewed as qualitative since the pre-buckling deformations were neglected, even though the zero bending moment conditions were satisfied approximately in the case of simply supported panels. The symmetric and antisymmetric shear buckling modes were not mentioned (see Stein and Neff [15]) and parameter studies on the aspect ratio and panel curvatures were not presented. It appears that shear buckling of cross-ply cylindrical panels has not been analyzed in the open literature. Vibrations of laminated cylindrical panels in the presence of in-plane shear loads have not been previously investigated either, even for the special case of isotropic homogeneous panels. The above shortcomings will be addressed in this paper.

This paper aims to analyze the effects of shear loads on buckling and vibrations of antisymmetric, equal-thickness, thin cross-ply cylindrical panels. The cylindrical panels are simply supported along all four edges and they are subjected to the possibilities of axial compression and lateral pressure. Although this type of laminated structure may exhibit bending-stretching coupling behavior, the prebuckling deformation due to shear load alone is zero. The effects of geometric parameters (aspect ratio and panel curvature) and material parameters (Young's modulus ratio and number of layers) are examined, incorporating the possibility of either symmetric or antisymmetric mode of buckling. Frequency vs shear load, axial compression vs shear load and external laterial pressure vs shear load interaction curves are plotted for various geometric parameters. Results for the important special case of laminated flat rectangular plates and isotropic homogeneous cylindrical panels are presented.

The analysis is based on a solution of Donnell-type linearized equilibrium and compatibility equations for laminated cylindrical shells, written in terms of an out-of-plane deflection and a stress function. A double-sine series solution is assumed for the deflection and the linearized compatibility equation is satisfied exactly by assuming a similar double-sine series for the stress function. The equilibrium equation is satisfied approximately by using a Galerkin procedure. Thus, the computed shear buckling load and frequencies are upper-bounds. The eigenvalue problem is found to decouple into two sets of homogeneous algebraic equations corresponding to the symmetric and antisymmetric modes respectively [16]. A series solution of up to 25 terms is attempted for the symmetric mode while 30 terms are used for the antisymmetric mode.

2. GOVERNING EQUATIONS AND ANALYSIS

The Donnell-type shallow-shell non-linear dynamic equilibrium and capability equations for laminated thin cylindrical panels, written in terms of an out-of-plane displacement W (positive outwards) and a stress function F, are

$$\rho W,_{\bar{t}\bar{t}} + L_{D^*}(W) + L_{B^*}(F) + (1/R)(F,_{XX}) = F,_{YY}W,_{XX} + F,_{XX}W_{YY} - 2F,_{XY}W,_{XY} \quad (1)$$

$$L_{A^*}(F) - L_{B^*}(W) - (1/R)(W,_{XX}) = (W,_{XY})^2 - W,_{XX}W,_{YY} \quad (2)$$

respectively. In the above, ρ is the mass of the shell per unit surface area, $\bar{t}$ is time, R is the radius, X and Y are the in-plane axial and circumferential coordinates and the linear differential operators $L_{A^*}(\ )$, $L_{B^*}(\ )$ and $L_{D^*}(\ )$ are defined by Tennyson *et al.* [17] and Hui [18, 19]. The following non-dimensional quantities are introduced:

$$w = W/h, \qquad F = (Eh^3)(f), \qquad (x, y) = (X, Y)/(Rh)^{1/2}$$

$$t = \bar{t}\omega_r, \qquad (\omega_r)^2 = Eh/(\rho R^2),$$

$$a^*_{ij} = (Eh)(A^*_{ij}), \qquad b^*_{ij} = B^*_{ij}/h, \qquad d^*_{ij} = D^*_{ij}/(Eh^3)$$

$$(\sigma, p, \tau) = [R/(Eh^2)](-N_x, -N_y, +N_{xy}) = (-f,_{yy}, -f,_{xx}, -f,_{xy}) \quad (3)$$

where E is Young's modulus which may be arbitrarily specified, h is the total thickness of the laminated plate, ω_r is the reference frequency and the axial load σ (positive for compression) lateral pressure p (positive for external pressure) and the shear load τ correspond to the membrane stress resultants N_x, N_y and N_{xy} respectively. Thus, the non-dimensional linearized equilibrium and compatibility equations for laminated cylindrical panels are, respectively,

$$w,_{tt} + L_{d^*}(w) + L_{b^*}(f) + f,_{xx} + \sigma w,_{xx} + pw,_{yy} = 2\tau w,_{xy} \quad (4)$$

$$L_{a^*}(f) - L_{b^*}(w) - w,_{xx} = 0. \quad (5)$$

In the present special case of antisymmetric cross-ply laminates, the linear differential operators are

$$L_{a^*}(\ ) = a^*_{22}(\ ),_{xxxx} + (2a^*_{12} + a^*_{66})(\ ),_{xxyy} + a^*_{11}(\ ),_{yyyy} \quad (6)$$

$$L_{b^*}(\ ) = b^*_{21}(\ ),_{xxxx} + (b^*_{11} + b^*_{22} - 2b^*_{66})(\ ),_{xxyy} + b^*_{12}(\ ),_{yyyy} \quad (7)$$

$$L_{d^*}(\ ) = d^*_{11}(\ ),_{xxxx} + (2)(d^*_{12} + 2d^*_{66})(\ ),_{xxyy} + d^*_{22}(\ ),_{yyyy}. \quad (8)$$

In anticipation of enforcing simply the supported boundary condition, it is desirable to write down the bending moment expressions (presented in dimensional form for clarity purposes).

$$M_x = B^*_{11}F_{,YY} - B^*_{21}F_{,XX} - D^*_{11}W_{,XX} - D^*_{12}W_{,YY} \tag{9}$$

$$M_y = B^*_{12}F_{,YY} - B^*_{22}F_{,XX} - D^*_{12}W_{,XX} - D^*_{22}W_{,YY}. \tag{10}$$

Since the cross-ply cylindrical panel is simple supported along all four edges, the out-of-plane deflection can be written in terms of the double-sine series,

$$w(x, y, t) = e^{i\omega \bar{t}} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} c_{mn} \sin(Mx) \sin(Ny) \tag{11}$$

In the above, ω is the frequency, L is the length, B is the curved distance between the two longitudinal edges and the wave numbers M and N are defined to be,

$$M = m\pi(Rh)^{1/2}/L = m\pi(B/L)/\theta_s$$
$$N = n\pi/\theta_s, \qquad \theta_s = B/(Rh)^{1/2} \tag{12}$$

where m and n are positive integers and θ_s is the simplified flatness parameter. Based on the above deflection shape, the stress function which satisfies the linearized compatibility equation exactly is found to be,

$$f(x, y, t) = e^{i\omega \bar{t}} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \alpha_{mn} \sin(Mx) \sin(Ny) \tag{13a}$$

where

$$\alpha_{mn} = \left(\frac{-M^2 + C_{b*}(M, N)}{C^*_a(M, N)} \right) c_{mn} \tag{13b}$$

where the functions $C_{a*}(M, N)$ and $C_{b*}(M, N)$ are defined to be,

$$C_{a*}(M, N) = a^*_{22}M^4 + (2a^*_{12} + a^*_{66})(M^2N^2) + a^*_{11}M^4 \tag{14}$$

$$C_{b*}(M, N) = b^*_{21}M^4 + (b^*_{11} + b^*_{22} - 2b^*_{66})(M^2N^2) + b^*_{12}N^4. \tag{15}$$

Thus, it can be seen that the zero bending moment boundary conditions are satisfied along all four edges. The above stress function satisfies the boundary conditions that the in-plane stress perpendicular to each edge is zero, that is,

$$f_{,yy}(x = 0 \quad \text{or} \quad x = \theta_s L/B) = 0 \tag{16a}$$

$$f_{,xx}(y = 0 \quad \text{or} \quad y = \theta_s) = 0. \tag{16b}$$

Further, the in-plane strain tangent to each edge is constant.

Substituting the deflection $w(x, y, t)$ and the stress function $f(x, y, t)$ into the linearized equilibrium equation, one obtains,

$$(1/\tau) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} [(\omega/\omega_r)^2 + \sigma M^2 + pN^2 - C^*(M, N)] c_{mn} \sin(Mx) \sin(Ny)$$
$$+ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (2MN) c_{mn} \cos(Mx) \cos(Ny) = 0 \tag{17}$$

where

$$C^*(M,N) = C_{a^*}(M,N) + \{[C_{b^*}(M,N) - M^2]^2/C_{a^*}(M,N)\} \tag{18}$$

$$C_{d^*}(M,N) = d^*_{11}M^4 + (2)(d^*_{12} + 2d^*_{66})(M^2N^2) + d^*_{22}N^4. \tag{19}$$

Applying the Galerkin procedure of multiplying the above equilibrium equation by $\sin(Sx)\sin(Qy)$, where (s and q are positive integers),

$$S = s\pi(B/L)\theta_s$$
$$Q = q\pi/\theta_s \tag{20, 21}$$

and then integrating over the shell surface area, one obtains a system of homogeneous algebraic equations of the form,

$$(1/\tau)(z_{sq})(c_{sq}) + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{(mnsq)(c_{mn})}{(m^2 - s^2)(n^2 - q^2)}\right) = 0. \tag{22a}$$

In the above, the summation is taken only for $m \neq s$ and $n \neq q$, $m + s = \text{odd}$ and $n + q = \text{odd}$ and the diagonal elements z_{sq} are defined to be,

$$z_{sq} = (L/B)(\theta_s)^2[(\omega/\omega_r)^2 + \sigma S^2 + pQ^2 - C_*(S,Q)]/32. \tag{22b}$$

Further, the following integrations formulae were used in deriving equations (22a, b).

$$\int_{x=0}^{\theta_s(L/B)} \sin(Sx)\cos(Mx)\,dx = \begin{cases}\left(\dfrac{-2s}{(m^2 - s^2)(\pi)}\right)\theta_s(L/B) & \text{if } m + s \text{ is odd} \\ 0 & \text{otherwise}\end{cases} \tag{23}$$

$$\int_{y=0}^{\theta_s} \sin(Qy)\cos(Ny)\,dy = \begin{cases}\left(\dfrac{-2q\theta_s}{(n^2 - q^2)(\pi)}\right) & \text{if } n + q \text{ is odd} \\ 0 & \text{otherwise.}\end{cases} \tag{24}$$

In the special case of an isotropic homogeneous cylindrical panel, the non-dimensional linearized equilibrium and compatibility equations are (using the previously defined non-dimensional scheme),

$$[1/(4c^2)](w_{,xxxx} + w_{,yyyy} + 2w_{xxyy}) + f_{,xx} + w_{,tt} + \sigma w_{,xx} + pw_{,yy} = 2\tau w_{,xy}$$
$$f_{,xxxx} + f_{,yyyy} + 2f_{,xxyy} = w_{,xx} \tag{25, 26}$$

where $c = [3(1 - v^2)]^{1/2}$ and v is Poisson's ratio. Thus, it can be seen that

$$a^*_{22} = (2a^*_{12} + a^*_{66}) = a^*_{11} = 1$$
$$b^*_{ij} = 0$$
$$d^*_{11} = (2)(d^*_{12} + 2d^*_{66}) = d^*_{22} = 1/(4c^2). \tag{27–29}$$

The final explicit expression relating the frequency and applied loads is again given by equation (22a, b) except that $C_*(S,Q)$ is replaced by,

$$C_*(S,Q) = [(S^2 + Q^2)^2/(4c^2)] + [S^4/(S^2 + Q^2)^2]. \tag{30}$$

In passing, the present non-dimensional shear load is related to the k parameter defined by Stein to be,

$$k(\text{Stein}) = (2c\theta_s/\pi)^2(\tau). \tag{31}$$

The above homogeneous system of equations can be written in terms of two uncoupled sets of algebraic equations which correspond to the symmetric and antisymmetric modes. The symmetric mode is specified by $s + q$ = even integer and the antisymmetric mode is specified by $s + q$ = odd integer. Thus, the values of (s, q) for the 25-term solution for the symmetric mode are (1, 1); (1, 3), (2, 2), (3, 1); (1, 5), (2, 4), (3, 3), (4, 2), (5, 1); (1, 7), (2, 6), ..., (7, 1); (1, 9), (2, 8), ..., (9, 1). Similarly, the values of (s, q) for the 30-term antisymmetric mode are (1, 2), (2, 1); (1, 4), (2, 3), (3, 2), (4, 1); (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1); (1, 8), (2, 7), ..., (8, 1); (1, 10), (2, 9), ..., (10, 1). These two sets of equations are presented in matrix form in Appendix A. The number of terms used in the series solution was permitted to vary in order to achieve the desired accuracy.

The resulting eigenvalue problem can be solved using the well known power method. It should be noted that the shifted technique [20] is needed in the power method since the largest eigenvalue occurs in positive and negative pair. In order to avoid numerical difficulties for small simplified flatness parameter (that is, θ_s being small and the cylindrical panel tends to become a flat rectangular plate), it is necessary to re-write the first term in equation (22a) as $(\theta_s)^2(z_{sq}/\bar{\tau})(c_{sq})$ where $\bar{\tau} = (\theta_s)^2(\tau)$. A convergence study shows that a larger number of terms (>30) in the series solution is needed for $L/B > 3.75$ or $\theta_s > 3.75$.

Finally, in the case of no in-plane shear load, the frequency, axial load and lateral pressure are related by,

$$(\omega/\omega_r)^2 + \sigma S^2 + pN^2 = C_*(M, N). \tag{32}$$

This, a one-term solution for the deflection mode will suffice and the fundamental frequency (or classical buckling load or lateral pressure) can be obtained from equation (32) by searching the smallest eigenvalue for all discrete values of the integers m and n.

3. DISCUSSION OF RESULTS

Graphical representations of the effects of shear loads vibration and buckling of antisymmetric cross-ply cylindrical panels are presented. Typical boron-epoxy material is chosen in the example problems where the material properties are

$$E_1/E_2 = 10, \qquad G_{12}/E_2 = 1/3, \qquad \nu_{12} = 0.22. \tag{33}$$

The effects of bending–stretching coupling on shear buckling and vibration of the above laminated structure are demonstrated by comparing the results for infinite number of layers (N = infinite) and two layers ($N = 2$). The two-layer cylindrical panel consists of a 90° inner and a 0° outer layer. By reversing these two layers, the results remain almost unchanged (too small to be plotted). The effects of changing the geometric and material parameters are examined.

Figure 1 shows a graph of the shear buckling load vs the aspect ratio for simply supported antisymmetric cross-ply cylindrical panel ($0 \le \theta_s \le 1$ and $\theta_s = 2$) where the material parameters are specified by equation (33). Defining the modified shear buckling load parameter,

$$\begin{aligned} \bar{\tau} &= (\tau)(\theta_s)^2 \qquad \text{for} \quad 0 \le \theta_s \le 1 \\ \bar{\tau} &= \tau \qquad\qquad \text{for} \quad \theta_s \ge 1 \end{aligned} \tag{34}$$

it is found that $\bar{\tau}$ is not affected by the simplified flatness parameter in the range $0 \le \theta_s \le 1$, at least within plotting accuracy. For aspect ratio between about 1.75 and 2.75 (and less than 0.6), the antisymmetric mode is critical (that is, of engineering significance) while the symmetric mode is critical for the remaining ranges of aspect ratio. Comparing the orthotropic (N = infinite) and the two-layer ($N = 2$) solution, significant reductions in the

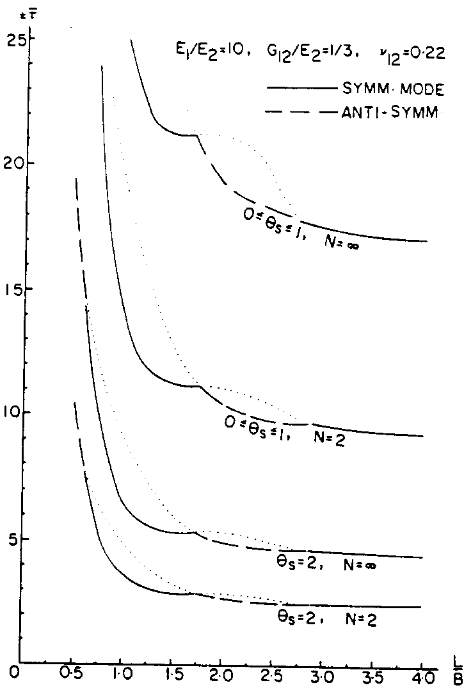

Fig. 1. Shear buckling load vs aspect ratio for simply supported antisymmetric cross-ply cylindrical panels.

shear buckling load (about 45%) are found as a result of the bending–stretching property of the laminated panels.

Figure 2 shows a graph of the modified shear buckling load vs the simplified flatness parameter for aspect ratio L/B being 1 and 2 and the number of layers N being 2 and infinite. The symmetric buckling mode is critical for square-shape cylindrical panel while the antisymmetric mode is critical for $L/B = 2.0$. Through the remaining figures, the shear buckling loads are found to be essentially unaffected by the simplified flatness parameter

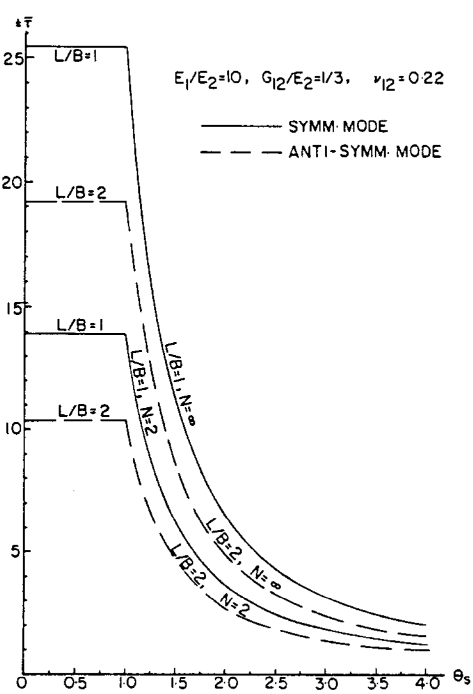

Fig. 2. Shear buckling load vs simplified flatness parameter for simply supported antisymmetric cross-ply cylindrical panels.

in the range $0 \leq \theta_s \leq 1$.

Figure 3 shows a plot of the frequency normalized with respect to the reference frequency vs the modified shear buckling load for ($\theta_s = 1$, $L/B = 2$) and ($\theta_s = 1.25$, $L/B = 1$). Both the symmetric and antisymmetric modes of vibration are involved in the particular critical interaction curve. Significant reductions in the frequency are found due to bending–stretching coupling. A sign change in the shear load will not affect the problem even in the two-layer case.

Figure 4 depicts the axial compression vs shear load interaction curves for simply supported antisymmetric cross-ply cylindrical panels for $\theta = \theta_s = 1$ and $\theta_s = 1.5$. The interaction curve for a particular geometry consists of a purely symmetric or a purely

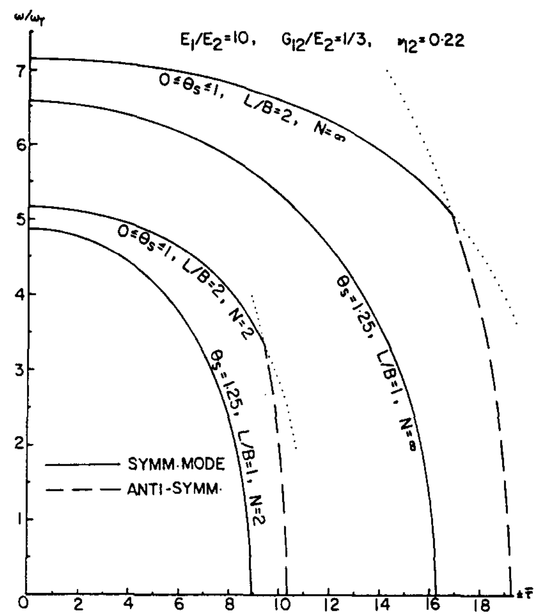

Fig. 3. Frequency–shear load interaction curves for simply supported antisymmetric cross-ply cylindrical panels.

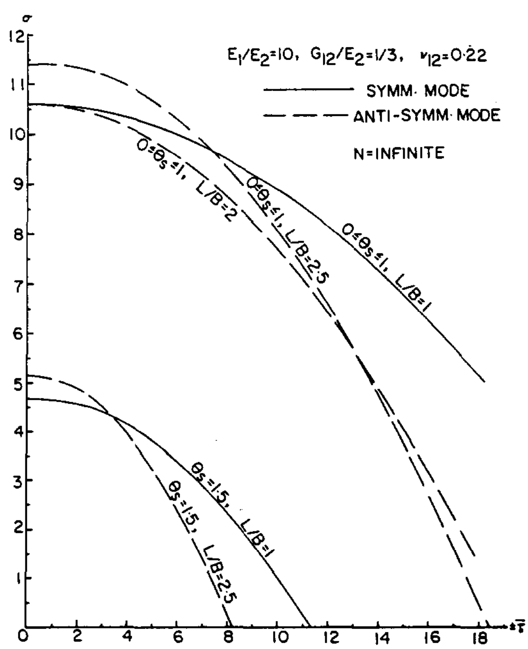

Fig. 4. Axial compression–shear load interaction curves for simply supported antisymmetric cross-ply cylindrical panels.

antisymmetric buckling mode. In the small simplified flatness parameter range ($0 \leq \theta_s \leq 1$), the axial buckling loads with no shear are the same for $L/B = 1$ (with $m = 1$) and $L/B = 2$ (with $m = 2$). However, the interaction curves for $L/B = 1$ and 2 are no longer the same in the presence of shear loads. In fact, the $L/B = 1$ curve is governed by the symmetric mode while the antisymmetric mode is critical for the $L/B = 2$ curve. The antisymmetric mode is critical in the range of aspect ratio L/B between 1.75 and 2.75 and a typical curve for $L/B = 2.5$ is plotted. Similar behavior can be found for larger values of the simplified flatness parameter and the curves for $\theta_s = 1.5$ are presented in this figure for $L/B = 1$ and 2.5.

Interaction curves between the external lateral pressure versus the shear buckling load are plotted in Fig. 5 for ($0 \leq \theta_s \leq 1, L/B = 2.5$) and ($\theta_s = 1.5$, $L/B = 1$ and 2.5). Unlike the axial compression vs shear interaction curves, this pressure–shear curve is governed by the symmetric mode for small values of $|\tau|$ and by the antisymmetric mode for larger values of $|\tau|$. Similar behavior is found for $\theta_s = 1.5$ and $L/B = 2.5$. For simplified flatness parameter outside the range ($1.75 \leq \theta_s \leq 2.75$), the interaction curves are governed by the symmetric mode alone (see the curve for $\theta_s = 1.5$, $L/B = 1.0$).

Finally, a graph of the modified shear load vs the Young's modulus ratio is presented for simply supported antisymmetric crossply cylindrical panels with $G_{12}/E_2 = 0.4$ and $\nu_{12} = 0.25$. The relation between the shear load and Young's modulus ratio for a particular geometry is found to be linear within plotting accuracy. The reduction of the shear buckling load due to bending–stretching coupling is quite pronounced by comparing the data for N = infinite and $N = 2$ laminated panels. No such reduction is found in the special case of an isotropic homogeneous cylindrical panel when $E_1/E_2 = 1.0$. The results for larger values of the simplified flatness parameter ($\theta_s = 1.5$ and $\theta_s = 2.0$) are plotted for square-shape ($L/B = 1$) cylindrical panel. The curves for $L/B = 2.5$ with $0 \leq \theta_s \leq 1$ are also presented for comparison purposes.

For the special case of isotropic homogeneous cylindrical panels, the modified shear buckling load vs simplified flatness parameter curves are plotted in Fig. 7 for various aspect ratios. The frequency–shear load interaction curves are shown in Fig. 8. As a check on the analysis, the present shear buckling loads agree with that presented by Batdorf *et al.* [11] and Stein and Neff [15] for some specified geometrical parameters.

4. CONCLUDING REMARKS

The effects of in-plane shear loads on vibration and buckling of simply supported antisymmetric cross-ply cylindrical panels have been investigated. It was found that

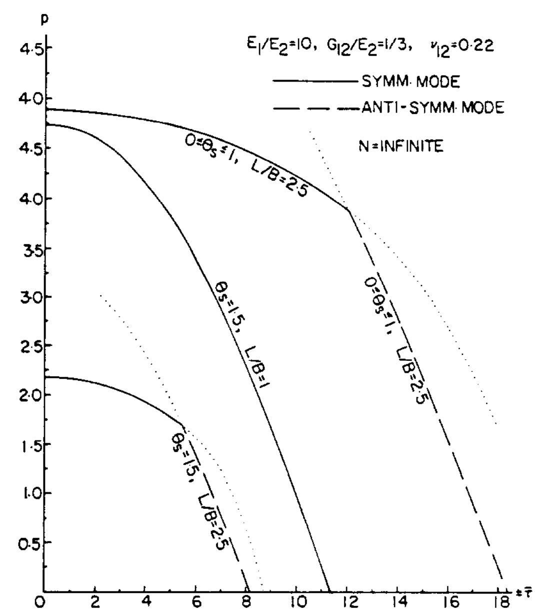

Fig. 5. External lateral pressure–shear load interaction curves for simply supported antisymmetric cross-ply cylindrical panels.

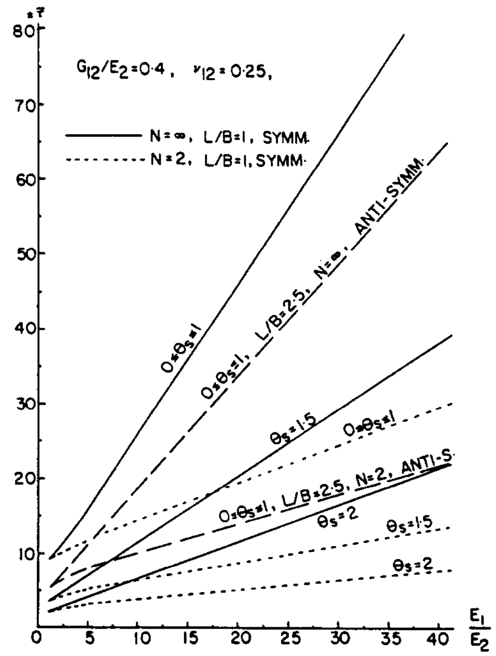

Fig. 6. Shear buckling load vs Young's modulus ratio for simply supported antisymmetric cross-ply cylindrical panels.

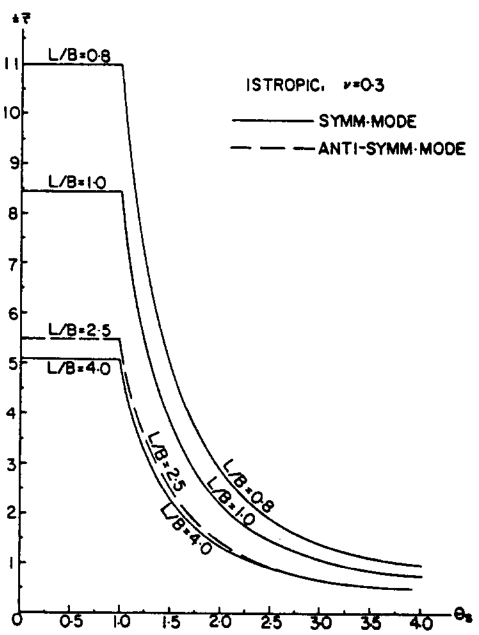

Fig. 7. Shear buckling load vs simplified flatness parameter for simply supported isotropic homogeneous cylindrical panels.

the eigenvalue problem decouples into two sets of algebraic homogeneous equations corresponding to the symmetric and antisymmetric modes respectively. Thus, care should be taken in computing the interaction curves to ensure that only the critical mode is retained. A concise geometric parameter variation (involving the aspect ratio and simplified flatness parameter) has been presented for boron-epoxy cross-ply panels. The results will allow the designers to obtain some essential preliminary estimates on the load carrying capacity of typical cross-ply cylindrical panels under shear loads, incorporating the possibility of axial compression and external lateral pressure.

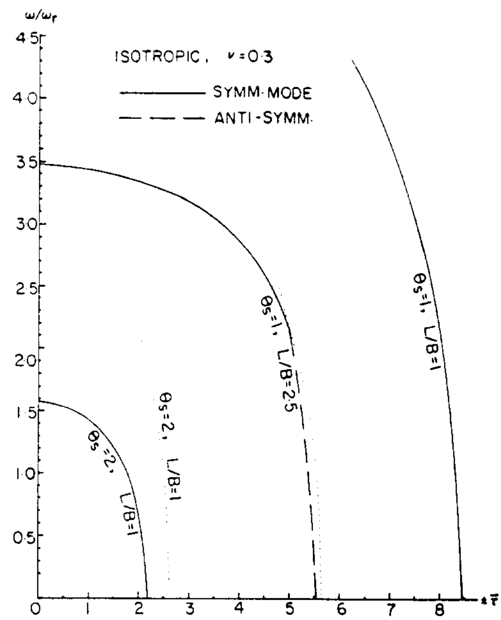

Fig. 8. Frequency–shear load interaction curves for simply supported isotropic homogeneous cylindrical panels.

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APPENDIX: THE SYMMETRIC AND ANTISYMMETRIC SETS OF EQUATIONS

The homogeneous system of algebraic equations for a 9-term symmetric mode approximation can be written in matrix form, $[A]x = 0$ where, $x = (c_{11}, c_{13}, c_{22}, c_{31}, c_{15}, c_{24}, c_{33}, c_{42}, c_{51})^{T}$ and

$$[A] = \begin{bmatrix} z_{11} & 0 & * & 0 & 0 & * & 0 & * & 0 \\ & z_{13} & * & 0 & 0 & * & 0 & * & 0 \\ & & z_{22} & * & * & 0 & * & 0 & * \\ & & & z_{31} & 0 & * & 0 & * & 0 \\ & & & & z_{15} & * & 0 & * & 0 \\ & & & & & z_{24} & * & 0 & * \\ & & \text{symmetric} & & & & z_{33} & * & 0 \\ & & \text{matrix} & & & & & z_{42} & * \\ & & & & & & & & z_{51} \end{bmatrix}. \tag{A1}$$

In the above matrix, a "*" represents a non-zero entry.

Using a 12-term approximation for the deflection mode, the homogeneous system of algebraic equations for the antisymmetric mode are $[B]x = 0$, where $x = (c_{12}, c_{21}, c_{14}, c_{23}, c_{41}, c_{16}, c_{25}, c_{34}, c_{43}, c_{52}, c_{61})^{T}$ and

$$[B] = \begin{bmatrix} z_{12} & * & 0 & * & 0 & * & 0 & * & 0 & * & 0 & * \\ & z_{21} & * & 0 & * & 0 & * & 0 & * & 0 & * & 0 \\ & & z_{14} & * & 0 & * & 0 & * & 0 & * & 0 & * \\ & & & z_{23} & * & 0 & * & 0 & * & 0 & * & 0 \\ & & & & z_{32} & * & 0 & * & 0 & * & 0 & * \\ & & & & & z_{41} & * & 0 & * & 0 & * & 0 \\ & & & & & & z_{16} & * & 0 & * & 0 & * \\ & \text{symmetric} & & & & & & z_{25} & * & 0 & * & 0 \\ & \text{matrix} & & & & & & & z_{34} & * & 0 & * \\ & & & & & & & & & z_{43} & * & 0 \\ & & & & & & & & & & z_{52} & * \\ & & & & & & & & & & & z_{61} \end{bmatrix} \tag{A2}$$